1 Inequalities and Modulus

DPP 1.1

Inequalities

Subjective Type

1. Solve $\frac{2x+3}{x^2+x-12} < \frac{1}{2}$.

2. Solve $\frac{1}{x-2} + \frac{1}{x-1} > \frac{1}{x}$.

3. Solve $\sqrt{8+2x-x^2} > 6-3x$.

4. Solve $\frac{\sqrt{2x-1}}{x-2} < 1$.

5. Solve $\sqrt{x-6} - \sqrt{10-x} \geq 1$.

6. Solve $\frac{1-\sqrt{21-4x-x^2}}{x+1} \geq 0$.

Single Correct Answer Type

7. If $f(x) = ax^2 + bx + c$ and $f(-1) \geq -4$, $f(1) \leq 0$ and $f(3) \geq 5$, then the least value of a is
   (a) 1/4 (b) 1/8 (c) 1/3 (d) −1/3

8. The complete set of values of x for which
$$\frac{x^3(x-1)^2(x+4)}{(x+1)(x-3)} \geq 0 \text{ is}$$
   (a) $(-\infty, -4] \cup (-1, 0] \cup (3, \infty) \cup \{1\}$
   (b) $(-\infty, -4] \cup (-1, 0] \cup (3, \infty) \cup \{-1\}$
   (c) $[-4, -1) \cup [0, 1) \cup (3, \infty)$
   (d) $[-4, -1) \cup [0, 1] \cup (3, \infty)$

9. The set of all values of x for which
$$\frac{(x+1)(x-3)^2(x-5)(x-4)^3(x-2)}{x} < 0 \text{ is}$$
   (a) $(-\infty, -1) \cup (0, 2) \cup (4, 5)$
   (b) $(-1, 0) \cup (2, 4) \cup (5, \infty)$
   (c) $(-1, 0) \cup (2, 3) \cup (4, 5)$
   (d) $(-\infty, -1) \cup (0, 2) \cup [3, 5)$

10. The solution set of inequality
$$\frac{(e^x-1)(2x-3)(x^2+x+2)}{(\sin x - 2)(x+1)x} \leq 0 \text{ is}$$
   (a) $\left[\frac{3}{2}, \infty\right)$ (b) $(-\infty, -1) \cup \left[\frac{3}{2}, \infty\right)$
   (c) $(-1, 0) \cup \left[\frac{3}{2}, \infty\right)$ (d) $R - \{0, -1\}$

11. The solution set of inequality $\frac{1}{2^x - 1} > \frac{1}{1 - 2^{x-1}}$ is
   (a) $(1, \infty)$ (b) $(0, \log_2(4/3))$
   (c) $(-1, \infty)$ (d) $(0, \log_2(4/3)) \cup (1, \infty)$

12. Let $A = \{x \mid x^2 - 4x + 3 < 0, x \in R\}$
$B = \{x \mid 2^{1-x} + p \leq 0; x^2 - 2(p+7)x + 5 \leq 0\}$

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If $B \subseteq A$, then $p \in$
(a) $[-4, -1]$ (b) $[-4, \infty)$
(c) $(-\infty, 1]$ (d) $[0, 1]$

13. Let $a, b > 0$ satisfies $a^3 + b^3 = a - b$. Then
(a) $a^2 + b^2 = 1$ (b) $a^2 + ab + b^2 < 1$
(c) $a^2 + b^2 > 1$ (d) none of these

Answers Key

Subjective Type

1. $x \in (-\infty, -4) \cup (-3, 3) \cup (6, \infty)$
2. $x \in (-\sqrt{2}, 0) \cup (1, \sqrt{2}) \cup (2, \infty)$
3. $x \in (1, 4]$
4. $x \in [1/2, 2) \cup (5, \infty)$
5. $x \in [(16 + \sqrt{7})/2, 10]$
6. $x \in [-2 - 2\sqrt{6}, -1) \cup [-2 + 2\sqrt{6}, 3]$

Single Correct Answer Type

7. (b) **8.** (a) **9.** (a) **10.** (b) **11.** (d)
12. (a) **13.** (b)

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DPP 1.2

Modulus

Subjective Type

1. Solve $|x|^2 - |x| + 4 = 2x^2 - 3|x| + 1$.
2. Solve the simultaneous equations $|x + 2| + y = 5, x - |y| = 1$.
3. Solve $x - \sqrt{1-|x|} < 0$.
4. Solve $|x - 1| - 2| = |x - 3|$.
5. Solve :$(|x - 1| - 3)(|x + 2| - 5) < 0$
6. For $a, b \in R$ prove that $\frac{|a+b|}{1+|a+b|} \le \frac{|a|}{1+|a|} + \frac{|b|}{1+|b|}$

Single Correct Answer Type

7. The number of integers satisfying $|2x - 3| + |x + 5| \le |x - 8|$ is
   (a) 5 (b) 6 (c) 7 (d) 8
8. Which of the following is not the solution of $|2x + 5| - |x - 3| \ge |x + 8|$?
   (a) $(-\infty, -8]$ (b) $[3, \infty)$
   (c) $[-8, 3)$ (d) none of these
9. The number of integral values of x satisfying the equation $|x - |x - 4|| = 4$ is
   (a) 5 (b) 7 (c) 9 (d) infinite
10. The solution of $|2x - 3| < |x + 2|$ is
    (a) $(-\infty, 1/3)$ (b) $(1/3, 5)$
    (c) $(5, \infty)$ (d) $(-\infty, 1/3) \cup (5, \infty)$
11. The solution of $\left|\frac{1}{x} - 2\right| < 4$ is
    (a) $(-\infty, -1/2)$ (b) $(1/6, \infty)$
    (c) $(-1/2, 1/6)$ (d) $(-\infty, -1/2) \cup (1/6, \infty)$
12. The solution of $\left|x + \frac{1}{x}\right| > 2$ is
    (a) $R - \{0\}$ (b) $R - \{-1, 0, 1\}$
    (c) $R - \{1\}$ (d) $R - \{-1, 1\}$
13. The solution of the inequality $\frac{|x+2| - |x|}{\sqrt{8 - x^3}} \ge 0$ is
    (a) $[-1, 2]$ (b) $[1, 2]$
    (c) $[-1, 1]$ (d) $[0, \sqrt[3]{4})$
14. If $\left|\frac{12x}{4x^2 + 9}\right| \le 1$, then
    (a) $x \in R$ (b) $x \in [-3, 3]$
    (c) $x \in [-1, \infty)$ (d) $x \in (-\infty, 2]$
15. Let a, b, c, d be real numbers such that $|a - b| = 2, |b - c| = 3, |c - d| = 4$. Then the sum of all possible values of $|a - d|$ is
    (a) 9 (b) 18 (c) 24 (d) 30
16. The number of solutions of the equation $\sqrt{x^2} - \sqrt{(x-1)^2} + \sqrt{(x-2)^2} = \sqrt{5}$ is
    (a) 0 (b) 1 (c) 2 (d) More than 2
17. If $|x^2 - 2x - 8| + |x^2 + x - 2| = 3|x + 2|$, then the set of all real values of x is
    (a) $[1, 4] \cup \{-2\}$ (b) $[1, 4]$
    (c) $[-2, 1] \cup [4, \infty)$ (d) $(-\infty, -2] \cup [1, 4]$
18. The number of integers satisfying the equation
    $|x| + \left|\frac{4 - x^2}{x}\right| = \left|\frac{4}{x}\right|$ is
    (a) 5 (b) 4 (c) 6 (d) 7
19. The equation $|2ax - 3| + |ax + 1| + |5 - ax| = \frac{1}{2}$ possesses
    (a) Infinite number of real solutions for some $a \in R$
    (b) finitely many real solutions for some $a \in R''$
    (c) no real solutions for some $a \in R$
    (d) no real solutions $a \in R$
20. The set of values of x satisfying $\left|\frac{x^2 - 5x + 4}{x^2 - 4}\right| \le 1$ is
    (a) $\left[0, \frac{8}{5}\right]$ (b) $\left[\frac{8}{5}, \infty\right]$
    (c) $[-\infty, -2]$ (d) R

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Answers Key

Subjective Type

1. $x = \pm 3$
2. $x = 2$
3. $x \in \left[-1, -\frac{1}{2} + \frac{\sqrt{5}}{2}\right)$
4. $[1, \infty)$
5. $x \in (-7, -2) \cup (3, 4)$

Single Correct Answer Type

7. (c)	**8.** (c)	**9.** (d)	**10.** (b)	**11.** (d)
12. (b)	**13.** (a)	**14.** (a)	**15.** (b)	**16.** (c)
17. (a)	**18.** (b)	**19.** (d)	**20.** (a)	

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CHAPTER

2

Theory of Equations

DPP 2.1

Equations Reducible to Quadratic, Identity, Factor and Remainder Theorem

Single Correct Answer Type

1. Number of real solutions of $\sqrt{2x-4}-\sqrt{x+5}=1$ is

(a) 0 (b) 1 (c) 2 (d) infinite

2. Number of real solutions of $\sqrt{x}+\sqrt{x-\sqrt{1-x}}=1$ is

(a) 0 (b) 1 (c) 2 (d) infinite

3. The set of real values of a for which the equation $\frac{2a^2+x^2}{a^3-x^3}-\frac{2x}{ax+a^2+x^2}+\frac{1}{x-a}=0$ has a unique solution is

(a) $(-\infty, 1)$ (b) $(-1, \infty)$

(c) $(-1, 1)$ (d) $R-\{0\}$

4. Number of distinct real solutions of the equation $x^2+\left(\frac{x}{x-1}\right)^2=8$ is

(a) 1 (b) 2 (c) 3 (d) 4

5. If m, n are positive integers and $m+n\sqrt{2}=\sqrt{41+24\sqrt{2}}$, then $(m+n)$ is equal to

(a) 5 (b) 6 (c) 7 (d) 8

6. The equation $(x+3-4(x-1)^{1/2})^{1/2}+(x+8-6(x-1)^{1/2})^{1/2}=1$ has

(a) no solution (b) only 1 solution

(c) only 2 solutions (d) more than 2 solutions

7. The number of solutions of $\sqrt{3x^2+x+5}=x-3$ is

(a) 0 (b) 1 (c) 2 (d) 4

8. The number of real or complex solutions of $x^2-6|x|+8=0$ is

(a) 6 (b) 7 (c) 8 (d) 9

9. If α, β are the roots the quadratic equation $x^2-(3+2^{\sqrt{\log_2 3}}-3^{\sqrt{\log_3 2}})x-2(3^{\log_3 2}-2^{\log_2 3})=0$, then the value of $\alpha^2+\alpha\beta+\beta^2$ is equal to

(a) 11 (b) 7 (c) 3 (d) 5

10. Which of the following is not true for equation $x^2 \log 8 - x \log 5 = 2(\log 2) - x$

(a) equation has one integral root

(b) equation has no irrational roots

(c) equation has rational roots

(d) none of these

11. Let $f(x)$ be a quadratic expression such that $f(-1)+f(2)=0$. If one root of $f(x)=0$ is 3, then the other root of $f(x)=0$ lies in

(a) $(-\infty, -3)$ (b) $(-3, \infty)$

(c) $(0, 5)$ (d) $(5, \infty)$

12. If $f(x)=(x^2+3x+2)(x^2-7x+a)$ and $g(x)=(x^2-x-12)(x^2+5x+b)$, then the value of a and b, if $(x+1)(x-4)$ is H.C.F. of $f(x)$ and $g(x)$ is

(a) $a=10 : b=6$ (b) $a=4 : b=12$

(c) $a=12 : b=4$ (d) $a=6 : b=10$

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13. The remainder obtained when the polynomial $x + x^3 + x^9 + x^{27} + x^{81} + x^{243}$ is divided by $x^2 - 1$ is

(a) $6x + 1$ (b) $5x + 1$

(c) $4x$ (d) $6x$

Comprehension Type

For Q. 14 and 15

Let $P(x)$ be a polynomial of degree at most 5 which leaves remainders -1 and 1 upon division by $(x - 1)^3$ and $(x + 1)^3$, respectively.

14. Number of real roots of $P(x) = 0$ is

(a) 1 (b) 3 (c) 5 (d) 2

15. The sum of pairwise product of all roots (real and complex) of $P(x) = 0$ is

(a) $-\frac{5}{3}$ (b) $-\frac{10}{3}$ (c) 2 (d) -5

For Q. 16 and 17

Let $f(x) = ax^2 + bx + c$, $a \neq 0$, $a, b, c \in I$. Suppose that $f(1) = 0$, $50 < f(7) < 60$ and $70 < f(8) < 80$.

16. The least value of $f(x)$ is

(a) 3/4 (b) 9/2 (c) $-9/8$ (d) 3/4

17. Number of integral values of x for which $f(x) < 0$ is

(a) 0 (b) 1 (c) 2 (d) 3

Answers Key

Single Correct Answer Type

1. (b)	**2.** (b)	**3.** (d)	**4.** (c)	**5.** (c)
6. (d)	**7.** (a)	**8.** (a)	**9.** (b)	**10.** (d)
11. (b)	**12.** (c)	**13.** (b)		

Comprehension Type

14. (a)	**15.** (b)
16. (c)	**17.** (b)

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DPP 2.2

Quadratic Equation and its Roots

Single Correct Answer Type

1. Let $f(x) = x^2 - ax + b$, 'a' is odd positive integer and the roots of the equation $f(x) = 0$ are two distinct prime numbers. If $a + b = 35$, then the value of $f(10) =$
   (a) –8 (b) –10 (c) –4 (d) 0
2. If $0 < \alpha < \beta < \gamma < \pi/2$, then the equation
   $(x - \sin\beta)(x - \sin\gamma) + (x - \sin\alpha)(x - \sin\gamma) + (x - \sin\alpha)(x - \sin\beta) = 0$ has
   (a) real and unequal roots.
   (b) non-real roots.
   (c) real and equal roots.
   (d) real and unequal roots greater than 2.
3. If the system of equation $r^2 + s^2 = t$ and $r + s + t = \dfrac{k-3}{2}$ has exactly one real solution, then the value of k is
   (a) 1 (b) 2 (c) 3 (d) 4
4. If $a, b, c \in R$ and $3b^2 - 8ac < 0$, then the equation $ax^4 + bx^3 + cx^2 + 5x - 7 = 0$ has
   (a) all real roots
   (b) all imaginary roots
   (c) exactly two real and two imaginary roots
   (d) none
5. For real solution of equation $\sqrt[3]{x + 3p + 1} - \sqrt[3]{x} = 1$, we have
   (a) $p \geq 1/4$ (b) $p \geq -1/4$
   (c) $p \geq 1/3$ (d) $p \geq -1/3$
6. For a, b, c non-zero, real distinct, the equation, $(a^2 + b^2)x^2 - 2b(a + c)x + b^2 + c^2 = 0$ has non-zero real roots. One of these roots is also the root of the equation:
   (a) $(b^2 - c^2)x^2 + 2a(b - c)x - a^2 = 0$
   (b) $(b^2 + c^2)x^2 - 2a(b + c)x + a^2 = 0$
   (c) $a^2x^2 + a(c - b)x - bc = 0$
   (d) $a^2x^2 - a(b - c)x + bc = 0$
7. The equation $x^2 + bx + c = 0$ has distinct roots. If 2 is subtracted from each root the result are the reciprocal of the original roots, then $b^2 + c^2$ is
   (a) 2 (b) 3 (c) 4 (d) 5
8. The equation $(x^2 + 3x + 4)^2 + 3(x^2 + 3x + 4) + 4 = x$ has
   (a) all its solutions real but not all positive
   (b) only two of its solutions real
   (c) two of its solutions positive and two negative
   (d) none of solutions real.

Multiple Correct Answers Type

9. If $c \neq 0$ and the equation $\dfrac{p}{2x} = \dfrac{a}{x+c} + \dfrac{b}{x-c}$ has two equal roots, then p can be
   (a) $(\sqrt{a} - \sqrt{b})^2$ (b) $(\sqrt{a} + \sqrt{b})^2$
   (c) $a + b$ (d) $a - b$
10. The equation $(ay - bx)^2 + 4xy = 0$ has rational solutions x, y for
    (a) $a = \dfrac{1}{2}, b = 2$ (b) $a = 4, b = \dfrac{1}{8}$
    (c) $a = 1, b = \dfrac{3}{4}$ (d) $a = 2, b = 1$
11. Let a, b, c and $m \in R^+$. The possible value of m (independent of a, b and c) for which atleast one of the following equations have real roots is

$$\left.\begin{aligned} ax^2 + bx + cm &= 0 \\ bx^2 + cx + am &= 0 \\ cx^2 + ax + bm &= 0 \end{aligned}\right\}$$

    (a) $\dfrac{1}{2}$ (b) $\dfrac{1}{8}$ (c) $\dfrac{1}{12}$ (d) $\dfrac{1}{4}$

Answers Key

Single Correct Answer Type

1. (a) 2. (a) 3. (b) 4. (c) 5. (b)
6. (c) 7. (d) 8. (d)

Multiple Correct Answers Type

9. (a, b) 10. (a, c) 11. (b, c, d)

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DPP 2.3

Relation Between Roots and Coefficient

Single Correct Answer Type

1. If one root of the equation $(x-1)(7-x)=m$ is three times the other, then m is equal to
(a) -5 (b) 0 (c) 2 (d) 5

2. If the roots of the equation $ax^2-4x+a^2=0$ are imaginary and the sum of the roots is equal to their product then a is
(a) -2 (b) 4 (c) 2 (d) Noneofthese

3. If the sum of squares of roots of equation $x^2-(\sin\alpha-2)x-(1+\sin\alpha)=0$ is the least, then α is equal to
(a) $\pi/4$ (b) $\pi/3$ (c) $\pi/2$ (d) $\pi/6$

4. If α and β are the roots of the equation $ax^2+bx+c=0$ then the sum of the roots of the equation $a^2x^2+(b^2-2ac)x+b^2-4ac=0$ is
(a) $-(\alpha^2-\beta^2)$ (b) $(\alpha+\beta)^2-2\alpha\beta$
(c) $\alpha^2\beta+\beta\alpha^2-4\alpha\beta$ (d) $-(\alpha^2+\beta^2)$

5. If the roots of the quadratic equation $ax^2+bx-b=0$, where $a, b\in R$ such that $a\cdot b>0$, are α and β, then the value of $\log_{|(\beta-1)|}|(\alpha-1)|$ is
(a) 1 (b) -1 (c) 0 (d) noneofthese

6. If $\cos^4\alpha+k$ and $\sin^4\alpha+k$ are the roots of $x^2+\lambda(2x+1)=0$ and $\sin^2\alpha+1$ and $\cos^2\alpha+1$ are the roots of $x^2+8x+4=0$, then the sum of the possible values of λ is
(a) 2 (b) -1 (c) 1 (d) 3

7. Let $f(x)=ax^2+bx+c$, $g(x)=ax^2+qx+r$, where $a, b, c, q, r\in R$ and $a<0$. If α, β are the roots of $f(x)=0$ and $\alpha+\delta, \beta+\delta$ are the roots of $g(x)=0$, then
(a) $f_{max.}>g_{max.}$
(b) $f_{max.}<g_{max.}$
(c) $f_{max.}=g_{max.}$
(d) cant say anything about relation between $f_{max.}$ and g_{max}

8. If a, b, c are in geometric progression and the roots of the equations $ax^2+2bx+c=0$ are α and β and those of $cx^2+2bx+a=0$ are γ and δ then
(a) $\alpha\neq\beta\neq\gamma\neq\delta$ (b) $\alpha\neq\beta$ and $\gamma\neq\delta$
(c) $a\alpha=a\beta=c\gamma=c\delta$ (d) $\alpha=\beta;\ \gamma\neq\delta$

9. If α, β are the roots of the equation $ax^2+bx+c=0$ and $S_n=\alpha^n+\beta^n$, then $aS_{n+1}+bS_n+cS_{n-1}=(n\geq 2)$
(a) 0 (b) $a+b+c$
(c) $(a+b+c)n$ (d) n^2abc

10. Let $f(x)=ax^2+bx+c$, $g(x)=ax^2+px+q$, where $a, b, c, q, p, \in R$ and $b\neq p$. If their discriminants are equal and $f(x)=g(x)$ has a root α, then
(a) α will be A.M. of the roots of $f(x)=0$, $g(x)=0$
(b) α will be G.M. of all the roots of $f(x)=0$, $g(x)=0$
(c) α will be A.M. of the roots of $f(x)=0$ or $g(x)=0$
(d) α will be G.M. of the roots of $f(x)=0$ or $g(x)=0$

11. If α and β be the roots of the equation $x^2+3x+1=0$, then the value of $\left(\frac{\alpha}{1+\beta}\right)^2+\left(\frac{\beta}{\alpha+1}\right)^2$ is equal to
(a) 18 (b) 19 (c) 20 (d) 21

12. The roots of the equation $a(b-2c)x^2+b(c-2a)x+c(a-2b)=0$ are, when $ab+bc+ca=0$
(a) $1, \frac{c(a-2b)}{a(b-2c)}$ (b) $\frac{c}{a}, \frac{a-2b}{b-2c}$
(c) $\frac{a-2b}{a-2c}, \frac{a-2b}{b-2c}$ (d) None of these

Comprehension Type

For Q. 13 to 15
Let α, β be two real numbers satisfying the following relations $\alpha^2+\beta^2=5$, $3(\alpha^5+\beta^5)=11(\alpha^3+\beta^3)$

13. Possible value of $\alpha\beta$ is
(a) 2 (b) $-\frac{10}{3}$ (c) -2 (d) $\frac{10}{3}$

14. Possible value of $\alpha+\beta$ is
(a) ± 2 (b) ± 3 (c) ± 1 (d) $\pm\sqrt{3}$

15. Quadratic equation having roots α and β is
(a) $x^2\pm x+2=0$ (b) $x^2\pm 3x-2=0$
(c) $x^2\pm\sqrt{3}x+2=0$ (d) none of these

Answers Key

Single Correct Answer Type

1. (c)	2. (c)	3. (c)	4. (d)	5. (b)
6. (c)	7. (c)	8. (c)	9. (a)	10. (a)
11. (a)	12. (a)			

Comprehension Type

13. (a) 14. (b) 15. (d)

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DPP 2.4

Common Roots

Single Correct Answer Type

1. If the equations $2x^2 - 7x + 1 = 0$ and $ax^2 + bx + 2 = 0$ have a common root, then
   (a) $a = 2, b = -7$ (b) $a = \frac{-7}{2}, b = 1$
   (c) $a = 4, b = -14$ (d) $a = -4, b = 1$
2. If α and β, α and γ, α and δ are the roots of the equations $ax^2 + 2bx + c = 0$, $2bx^2 + cx + a = 0$ and $cx^2 + ax + 2b = 0$ respectively where a, b, c are positive real numbers, then $\alpha + \alpha^2$ is equal to
   (a) -1 (b) 1 (c) 0 (d) abc
3. The product of uncommon real roots of the equation $x^4 + 2x^3 - 8x^2 - 6x + 15 = 0$ and $x^3 + 4x^2 - x - 10 = 0$ is
   (a) -6 (b) -5 (c) 5 (d) 6
4. Number of values of x satisfying the pair of quadratic equations $x^2 - px + 20 = 0$ and $x^2 - 20x + p = 0$ for some $p \in R$ is
   (a) 0 (b) 1 (c) 2 (d) 3
5. If the equations $4x^2 - x - 1 = 0$ and $3x^2 + (\lambda + \mu)x + \lambda - \mu = 0$ have a root common, then the rational values of λ and μ are
   (a) $\lambda = 0, \mu = \frac{-3}{4}$ (b) $\lambda = \frac{-3}{4}, \mu = \frac{3}{4}$
   (c) $\lambda = \frac{-3}{4}, \mu = 0$ (d) $\lambda = \frac{-3}{4}, \mu = \frac{1}{4}$
6. If the equations $x^2 + 2\lambda x + \lambda^2 + 1 = 0$, $\lambda \in R$ and $ax^2 + bx + c = 0$, where a, b, c are lengths of sides of triangle have a common root, then the possible range of values of λ is
   (a) $(0, 2)$ (b) $(\sqrt{3}, 3)$
   (c) $(2\sqrt{2}, 3\sqrt{2})$ (d) $(0, \infty)$
7. If both the roots of $k(6x^2 + 3) + rx + 2x^2 - 1 = 0$ and $6k(2x^2 + 1) + px + 4x^2 - 2 = 0$ are common, then $2r - p$ is equal to
   (a) -1 (b) 0 (c) 1 (d) 2
8. $x^3 + 5x^2 + px + q = 0$ and $x^3 + 7x^2 + px + r = 0$ have two roots in common. If their third roots are γ_1 and γ_2, respectively, then $|\gamma_1 + \gamma_2| =$
   (a) 10 (b) 12 (c) 13 (d) 42
9. Let $a, b \in N$, $a \neq b$ and the two quadratic equations $(a-1)x^2 - (a^2+2)x + a^2 + 2a = 0$ and $(b-1)x^2 - (b^2+2)x + (b^2 + 2b) = 0$ have a common root. The value of ab is
   (a) 4 (b) 6 (c) 8 (d) 10
10. A quadratic equation $p(x) = 0$ having coefficient x^2 unity is such that $p(x) = 0$ and $p(p(p(x))) = 0$ have a common root, then
    (a) $p(0)\,p(1) > 0$ (b) $p(0)\,p(1) < 0$
    (c) $p(0)\,p(1) = 0$ (d) $p(0) = 0$ and $p(1) = 0$
11. If $ax^2 + bx + c = 0$ and $cx^2 + bx + a = 0$ $(a, b, c \in R)$ have a common non-real root, then which of the following is not true?
    (a) $-2|a| < b < |a|$ (b) $-2|c| < b < 2|c|$
    (c) $a = c$ (d) None of these
12. Let $P = \{x \mid x^2 + (m-1)x - 2(m+1) = 0, x \in R\}$ and $Q = \{x \mid (m-1)x^2 + mx + 1 = 0, x \in R\}$ Number of values of m such that $P \cup Q$ has exactly 3 distinct elements is
    (a) 5 (b) 6 (c) 7 (d) 8

Comprehension Type

For Q. 13 to 15

Consider quadratic equations $x^2 - ax + b = 0$ (i)
and $x^2 + px + q = 0$ (ii)

13. If the above equations have one common root and the other roots are reciprocals of each other, then $(q-b)^2$ equals
    (a) $bq(p-a)^2$ (b) $b(p-a)^2$
    (c) $q(p-a)^2$ (d) None of these
14. If for the equations (i) and (ii), one root is common and the equation (ii) have equal roots, then $b + q$ is equal to
    (a) $-ap$ (b) ap (c) $\frac{1}{2}ap$ (d) $2ap$
15. α, β are the roots of (i) and γ, δ are the roots of (ii). $\alpha, \beta, \gamma, \delta$ are in A.P. Then $q - b$ equals
    (a) $p^2 - a^2$ (b) $a^2 - p^2$
    (c) $\frac{p^2 - a^2}{4}$ (d) None of these

Answers Key

Single Correct Answer Type

1. (c)	2. (a)	3. (d)	4. (d)	5. (c)
6. (a)	7. (b)	8. (b)	9. (c)	10. (c)
11. (d)	12. (c)			

Comprehension Type

13. (a) 14. (c) 15. (c)

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DPP 2.5

Quadratic Function and Location of Roots

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Single Correct Answer Type

1. If the equation $|x^2 - 5x + 6| - \lambda x + 7\lambda = 0$ has exactly three solutions, then λ is equal to

(a) $-7 + \sqrt{23}$ (b) $-9 + 4\sqrt{5}$

(c) $-7 - \sqrt{23}$ (d) $-9 - 4\sqrt{5}$

2. Let α, β $(a < b)$ be the roots of the equation $ax^2 + bx + c = 0$. If $\lim_{x \to m} \frac{|ax^2 + bx + c|}{ax^2 + bx + c} = 1$, then

(a) $\frac{|a|}{a} = -1, m < \alpha$ (b) $a > 0, \alpha < m < \beta$

(c) $\frac{|a|}{a} = 1, m > \beta$ (d) $a < 0, m > \beta$

3. If the quadratic polynomials defined on real coefficients

$$P(x) = a_1x^2 + 2b_1x + c_1 \text{ and } Q(x) = a_2x^2 + 2b_2x + c_2$$

take positive values $\forall\, x \in R$, what can we say for the trinomial $g(x) = a_1a_2x^2 + b_1b_2x + c_1c_2$?

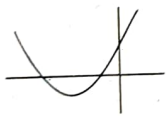
(a) $g(x)$ takes positive values only.

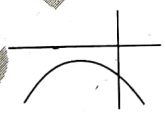
(b) $g(x)$ takes negative values only.

(c) $g(x)$ can take positive as well as negative values.

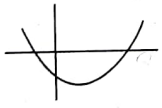
(d) Nothing definite can be said about $g(x)$.

4. For which of the following graphs the quadratic expression $y = ax^2 + bx + c$ the product abc is negative?

(a) 

(b) 

(c)

(d) 

5. The difference of maximum and minimum value of $\frac{x^2 + 4x + 9}{x^2 + 9}$ is

(a) 1/3 (b) 2/3 (c) –2/3 (d) 4/3

6. If $a > 1$, then the roots of the equation $(1 - a)x^2 + 3ax - 1 = 0$ are

(a) one positive and one negative

(b) both negative

(c) both positive

(d) both non real complex

7. The values of 'a' for which the quadratic expression $ax^2 + (a - 2)x - 2$ is negative for exactly two integral values of x, belongs to

(a) $[-1, 1]$ (b) $[1, 2)$

(c) $[-1, 1]$ (d) $[-2, -1)$

8. The values of a for which the roots of the equation $(a + 1)x^2 - 3ax + 4a = 0$ $(a \neq -1)$ are real and greater than 1 are

(a) $\left[-\frac{10}{7}, 1\right]$ (b) $\left[-\frac{12}{7}, 0\right]$

(c) $\left[-\frac{16}{7}, -1\right)$ (d) $\left(-\frac{16}{7}, 0\right)$

9. The equation $ax^4 - 2x^2 - (a - 1) = 0$ will have real and unequal roots if

(a) $0 < a < 1$ (b) $a > 0, a \neq 1$

(c) $a < 0, a \neq 1$ (d) none of these

10. If $ax^2 + bx + c = 0$, $a \neq 0$, $a, b, c \in R$ has distinct real roots in (1, 2), then a and $5a + 2b + c$ have

(a) same sign (b) opposite sign

(c) not determined (d) none of these

11. If $c < a < b < d$, then roots of the equation

$$bx^2 + (1 - b(c + d))x + bcd - a = 0$$

(a) are real and one lies between c and a

(b) are real and distinct in which one lies between a and b

(c) are real and distinct in which one lies between c and d

(d) are not real

12. If $2a$, b, $2c$ are in A.P., where a, b, c are R^+, then the expression $f(x) = (ax^2 - bx + c)$ has

(a) both roots negative

(b) both roots positive

(c) atleast one root between 0 and 2

(d) roots are of opposite sign.

13. If a, b, c are positive numbers such that $a > b > c$ and the equation $(a + b - 2c)x^2 + (b + c - 2a)x + (c + a - 2b) = 0$ has a root in the interval $(-1, 0)$, then

(a) b cannot be the G.M. of a, c

(b) b may be the G.M. of a, c

(c) b is the G.M. of a, c

(d) none of these

14. If the quadratic equation $x^2 - 36x + \lambda = 0$ has roots α and β such that $\alpha, \beta \in N$ and $\frac{\lambda}{5} \notin Z$ and λ assumes minimum possible value then $\frac{\sqrt{\alpha+2}\sqrt{\beta+2}}{|\alpha-\beta|}$ is equal to

(a) $\frac{3}{8}$ (b) $\frac{3}{16}$ (c) $\frac{\sqrt{111}}{34}$ (d) $\frac{\sqrt{111}}{17}$

15. If the equation $2^{2x} + a \cdot 2^{x+1} + a + 1 = 0$ has roots of opposite sign, then the exhaustive set of real values of a is

(a) $(-\infty, 0)$ (b) $\left(-1, \frac{-2}{3}\right)$

(c) $\left(-\infty, \frac{-2}{3}\right)$ (d) $(-1, \infty)$

Answers Key

Single Correct Answer Type

1. (b) **2.** (c) **3.** (a) **4.** (b) **5.** (d)
6. (c) **7.** (b) **8.** (c) **9.** (a) **10.** (a)
11. (c) **12.** (b) **13.** (a) **14.** (a) **15.** (b)

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DPP 2.6

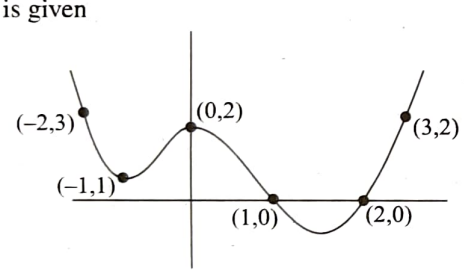
Higher Order Equations

Single Correct Answer Type

1. Let a, b, c be three distinct non-zero real numbers satisfying the system of equation $\frac{1}{a}+\frac{1}{a-1}+\frac{1}{a-2}=1$; $\frac{1}{b}+\frac{1}{b-1}+\frac{1}{b-2}=1$; $\frac{1}{c}+\frac{1}{c-1}+\frac{1}{c-2}=1$. Then abc =

(a) 1 (b) 2 (c) 3 (d) 4

2. In the given figure graph of $y = p(x) = x^4 + ax^3 + bx^2 + cx + d$ is given



The product of all imaginary roots of $p(x) = 0$ is

(a) 1 (b) 2 (c) 1/3 (d) 1/4

3. If $a^3 - 3a^2 + 5a - 17 = 0$ and $b^3 - 3b^2 + 5b + 11 = 0$ are such that $a + b$ is a real number, then the value of $a + b$ is

(a) –1 (b) 1 (c) 2 (d) –2

4. Let $f(x) = x^4 + ax^3 + bx^2 + cx + d$ be a polynomial with real coefficients and real zeroes. If $|f(i)| = 1$, (where $i = \sqrt{-1}$) then $a + b + c + d$ is equal to

(a) –1 (b) 1

(c) 0 (d) can't be determined

5. If $f(x)$ is a polynomial of degree four with leading coefficient one satisfying $f(1) = 1, f(2) = 2, f(3) = 3$ then $\left[\frac{f(-1)+f(5)}{f(0)+f(4)}\right]$ (where [.] represent greatest integer function) is equal to

(a) 4 (b) 5 (c) 6 (d) 7

6. Let $P(x) = x^6 - x^5 - x^3 - x^2 - x$ and $Q(x) = x^4 - x^3 - x^2 - 1$. If α, β, γ and δ are the roots of $Q(x) = 0$, then the value of $P(\alpha) + P(\beta) + P(\gamma) + P(\delta)$ is equal to

(a) 4 (b) 6 (c) 8 (d) 12

7. The line $y = mx + 1$ touches the curves $y = -x^4 + 2x^2 + x$ at two points $P(x_1, y_1)$ and $Q(x_2, y_2)$. The value of $x_1^2 + x_2^2 + y_1^2 + y_2^2$ is

(a) 4 (b) 6 (c) 8 (d) 10

8. If $a + b + c = 24$, $a^2 + b^2 + c^2 = 210$, $abc = 440$. Then the least value of $a - b - c$ is

(a) –2 (b) 2 (c) 8 (d) –14

9. If the roots of $x^4 + qx^2 + kx + 225 = 0$ are in arithmetic progression, then the value of q, is

(a) 15 (b) 25 (c) 35 (d) –50

Multiple Correct Answers Type

10. If α, β, γ are the roots of the equation $9x^3 - 7x + 6 = 0$ then the equation $x^3 + Ax^2 + Bx + C = 0$ has roots $3\alpha + 2$, $3\beta + 2$, $3\gamma + 2$, where

(a) $A = 6$ (b) $B = -5$

(c) $C = 24$ (d) $A + B + C = 23$

11. Let 'm' be a real number, and suppose that two of the three solutions of the cubic equation $x^3 + 3x^2 - 34x = m$ differ by 1. Then possible value of 'm' is/are

(a) 120 (b) 80 (c) –48 (d) –32

12. Let $f(x) = x^3 + x + 1$, let $p(x)$ be a cubic polynomial such that the roots of $p(x) = 0$ are the squares of the roots of $f(x) = 0$, then

(a) $p(1) = 3$

(b) the value of $P(n)$, $n \in N$ is odd

(c) Sum of all roots of $p(x) = 0$ is –2

(d) Sum of product of roots taken two at a time is 1

Comprehension Type

For Q. 13 and 14

The polynomial $P(x) = x^3 + ax^2 + bx + c$ has the property that the mean of its roots, the product of its roots, and the sum of its coefficients are all equal. If the y-intercept of the graph of $y = P(x)$ is 2,

13. The value of b is

(a) –11 (b) –9 (c) –7 (d) 5

14. The value of $P(1)$ is

(a) 0 (b) –1 (c) 2 (d) –2

Answers Key

Single Correct Answer Type

1. (b) **2.** (a) **3.** (c) **4.** (c) **5.** (b)
6. (b) **7.** (b) **8.** (d) **9.** (d)

Multiple Correct Answers Type

10. (c, d) **11.** (a, c) **12.** (a, b, c, d)

Comprehension Type

13. (a) **14.** (d)

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CHAPTER

3

Complex Numbers

DPP 3.1

Complex Number, Definition

Single Correct Answer Type

1. The value of $\sum_{n=0}^{100} i^{n!}$ equals (where $i = \sqrt{-1}$)

   (a) -1 (b) i (c) $2i + 95$ (d) $97 + i$

2. Suppose n is a natural number such that $|i + 2i^2 + 3i^3 + \ldots + ni^n| = 18\sqrt{2}$, where i is the square root of -1. Then n is

   (a) 9 (b) 18 (c) 36 (d) 72

3. Let $i = \sqrt{-1}$ define a sequence of complex number by $z_1 = 0, z_{n+1} = z_n^2 + i$ for $n \geq 1$. In the complex plane, then z_{111} lies in which quadrant?

   (a) 1 (b) 2 (c) 3 (d) 4

4. The complex number, $z = \dfrac{(-\sqrt{3} + 3i)(1 - i)}{(3 + \sqrt{3}i)(i)(\sqrt{3} + \sqrt{3}i)}$

   (a) lies on real axis
   (b) lies on imaginary axis
   (c) lies in first quadrant
   (d) lies in second quadrant

5. a, b, c are positive numbers in G.P. and the equation $(a + di)x^2 + 2(b + ei)x + (c + if) = 0$ have one real root. Then $\dfrac{a}{d}, \dfrac{b}{e}, \dfrac{c}{f}$ are in $(a, b, c, d, e, f \in R)$

   (a) A.P. (b) G.P.
   (c) H.P. (d) None of these

6. The equation $Z^3 + iZ - 1 = 0$ has

   (a) three real roots (b) one real root
   (c) no real roots (d) no real or complex roots

7. If a, b are complex numbers and one of the roots of the equation $x^2 + ax + b = 0$ is purely real whereas the other is purely imaginary, and $a^2 - \bar{a}^2 = kb$, then k is

   (a) 2 (b) 4 (c) 6 (d) 8

8. If Z is a non-real complex number, then the minimum value of $\dfrac{\text{Im } Z^5}{\text{Im}^5 Z}$ is

   (a) -1 (b) -2 (c) -4 (d) -5

9. If z_1, z_2, z_3 are any three complex numbers on Argand plane, then $z_1 (\text{Im}(\bar{z}_2 z_3)) + z_2 (\text{Im}\, \bar{z}_3 z_1)) + z_3 (\text{Im}\, \bar{z}_1 z_2))$ is equal to

   (a) 0 (b) $z_1 + z_2 + z_3$
   (c) $z_1 z_2 z_3$ (d) $\left(\dfrac{z_1 + z_2 + z_3}{z_1 z_2 z_3}\right)$

Multiple Correct Answers Type

10. Complex numbers whose real and imaginary parts x and y are integers and satisfy the equation $3x^2 - |xy| - 2y^2 + 7 = 0$

    (a) do not exist
    (b) exist and have equal modulus
    (c) form two conjugate pairs
    (d) do not form conjugate pairs

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11. If all the three roots of $az^3 + bz^2 + cz + d = 0$ have negative real parts ($a, b, c \in R$), then

(a) $ab > 0$ (b) $bc > 0$ (c) $ad > 0$ (d) $bc - ad > 0$

12. Suppose three real numbers a, b, c are in GP. Let $z = \dfrac{a + ib}{c - ib}$. Then

(a) $z = \dfrac{ib}{c}$ (b) $z = \dfrac{ia}{b}$

(c) $z = \dfrac{ia}{c}$ (d) $z = 0$

Answers Key

Single Correct Answer Type

1. (c) **2.** (c) **3.** (b) **4.** (b) **5.** (c)
6. (c) **7.** (b) **8.** (c) **9.** (a)

Multiple Correct Answers Type

10. (b, c) **11.** (a, b, c)
12. (a, b)

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DPP 3.2

Polar Form and Euler's Form of Complex Number

Single Correct Answer Type

1. The modulus and amplitude of $\frac{1+2i}{1-(1-i)^2}$ are

   (a) $\sqrt{2}$ and $\frac{\pi}{6}$ (b) 1 and $\frac{\pi}{4}$

   (c) 1 and 0 (d) 1 and $\frac{\pi}{3}$

2. The argument of $(z-a)(\bar{z}-b)$ is equal to that of $\frac{(\sqrt{3}+i)(1+\sqrt{3}i)}{1+i}$, where a, b are real numbers. If locus of z in the argand diagram is a circle with its centre at $(3+i)/2$, then the ordered pair (a, b) is

   (a) (3, 2) (b) (2, 1) (c) (2, 3) (d) (2, 4)

3. If a complex number z satisfies $|z|^2 + \frac{4}{|z|^2} - 2\left(\frac{z}{\bar{z}} + \frac{\bar{z}}{z}\right) - 16 = 0$, then the maximum value of $|z|$ is

   (a) $\sqrt{6}+1$ (b) 4 (c) $2+\sqrt{6}$ (d) 6

4. If $\cos\alpha + \cos\beta + \cos\gamma = 0 = \sin\alpha + \sin\beta + \sin\gamma$, then $\frac{\sin 3\alpha + \sin 3\beta + \sin 3\gamma}{\sin(\alpha+\beta+\gamma)}$ is equal to

   (a) 1 (b) –1 (c) 3 (d) –3

5. The least value of $|z-3-4i|^2 + |z+2-7i|^2 + |z-5+2i|^2$ occurs when

   (a) $1+3i$ (b) $3+3i$

   (c) $3+4i$ (d) None of these

6. The roots of the equation $x^4 - 2x^2 + 4 = 0$ are the vertices of a:

   (a) square inscribed in a circle of radius 2.

   (b) rectangle inscribed in a circle of radius 2.

   (c) square inscribed in a circle of radius $\sqrt{2}$.

   (d) rectangle inscribed in a circle of radius $\sqrt{2}$.

7. If z_1, z_2 are complex numbers such that $\text{Re}(z_1) = |z_1 - 2|$, $\text{Re}(z_2) = |z_2 - 2|$ and $\arg(z_1 - z_2) = \pi/3$, then $\text{Im}(z_1 + z_2) =$

   (a) $2/\sqrt{3}$ (b) $4/\sqrt{3}$ (c) $2/\sqrt{3}$ (d) $\sqrt{3}$

8. If $z = e^{2\pi i/5}$, then $1 + z + z^2 + z^3 + 5z^4 + 4z^5 + 4z^6 + 4z^7 + 4z^8 + 5z^9 =$

   (a) 0 (b) $4z^3$

   (c) $5z^4$ (d) $-4z^2$

9. If $z = (3+7i)(a+ib)$, where $a, b \in Z - \{0\}$, is purely imaginary, then minimum value of $|z|^2$ is

   (a) 74 (b) 45 (c) 65 (d) 58

Answers Key

Single Correct Answer Type

1. (c) **2.** (b) **3.** (c) **4.** (a) **5.** (d)
6. (d) **7.** (b) **8.** (c) **9.** (d)

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DPP 3.3

Properties of Modulus and Conjugate

Single Correct Answer Type

1. Let z be a complex number satisfying $|z + 16| = 4|z + 1|$. Then
(a) $|z| = 4$ (b) $|z| = 5$
(c) $|z| = 6$ (d) $3 < |z| < 68$

2. If $|z| = 1$ and $z' = \dfrac{1+z^2}{z}$, then
(a) z' lie on a line not passing through origin
(b) $|z'| = \sqrt{2}$
(c) Re $(z') = 0$
(d) Im $(z') = 0$

3. a, b, c are three complex numbers on the unit circle $|z| = 1$, such that $abc = a + b + c$. Then $|ab + bc + ca|$ is equal to
(a) 3 (b) 6 (c) 1 (d) 2

4. If z_1, z_2, z_3 are complex numbers such that $|z_1| = |z_2| = |z_3| = 1$, then $|z_1 - z_2|^2 + |z_2 - z_3|^2 + |z_3 - z_1|^2$ cannot exceed
(a) 6 (b) 9 (c) 12 (d) none of these

5. Number of ordered pairs(s) (a, b) of real numbers such that $(a + ib)^{2008} = a - ib$ holds good is
(a) 2008 (b) 2009 (c) 2010 (d) 1

6. The region represented by the inequality $|2z - 3i| < |3z - 2i|$ is
(a) the unit disc with its centre at $z = 0$
(b) the exterior of the unit circle with its centre at $z = 0$
(c) the interior of a square of side 2 units with its centre at $z = 0$
(d) none of these

7. If ω is any complex number such that $z\omega = |z|^2$ and $|z - \bar{z}| + |\omega + \bar{\omega}| = 4$, then as ω varies, then the area bounded by the locus of z is
(a) 4 sq. units (b) 8 sq. units
(c) 16 sq. units (d) 12 sq. units

8. If $az^2 + bz + 1 = 0$; where $a, b, z \in C$; $|a| = \dfrac{1}{2}$; and have a root α such that $|\alpha| = 1$, then $|a\bar{b} - b| =$
(a) 1/4 (b) 1/2 (c) 5/4 (d) 3/4

9. Let p and q are complex numbers such that $|p| + |q| < 1$. If z_1 and z_2 are the roots of the $z^2 + pz + q = 0$, then which one of the following is correct?
(a) $|z_1| < 1$ and $|z_2| < 1$
(b) $|z_1| > 1$ and $|z_2| > 1$
(c) If $|z_1| < 1$, then $|z_2| > 1$ and vice versa
(d) Nothing definite can be said

10. If z and w are two complex numbers simultaneously satisfying the equations, $z^3 + w^5 = 0$ and $z^2 . \bar{w}^4 = 1$, then
(a) z and w both are purely real
(b) z is purely real and w is purely imaginary
(c) w is purely real and z is purely imaginary
(d) z and w both are imaginary

11. All complex numbers 'z' which satisfy the relation $|z - |z + 1|| = |z + |z - 1||$ on the complex plane lie on the
(a) $y = x$
(b) $y = -x$
(c) circle $x^2 + y^2 = 1$
(d) line $x = 0$ or on a line segment joining $(-1, 0)$ to $(1, 0)$

12. If z_1, z_2 are two complex numbers such that $\left|\dfrac{z_1 - z_2}{z_1 + z_2}\right| = 1$ and $i z_1 = Kz_2$, where $K \in R$, then the angle between $z_1 - z_2$ and $z_1 + z_2$ is
(a) $\tan^{-1}\left(\dfrac{2K}{K^2+1}\right)$ (b) $\tan^{-1}\left(\dfrac{2K}{1-K^2}\right)$
(c) $-2\tan^{-1} K$ (d) $2\tan^{-1} K$

Multiple Correct Answers Type

13. w_1, w_2 be roots of $(a + \bar{c})z^2 + (b + \bar{b})z + (\bar{a} + c) = 0$. If $|z_1| < 1, |z_2| < 1$, then
(a) $|w_1| < 1$ (b) $|w_1| = 1$
(c) $|w_2| < 1$ (d) $|w_2| = 1$

14. A complex number z satisfies the equation $|Z^2 - 9| + |Z^2| = 41$, then the true statements among the following are
(a) $|Z + 3| + |Z - 3| = 10$
(b) $|Z + 3| + |Z - 3| = 8$
(c) Maximum value of $|Z|$ is 5
(d) Maximum value of $|Z|$ is 6

15. Let a, b, c be distinct complex numbers with $|a| = |b| = |c| = 1$ and z_1, z_2 be the roots of the equation $az^2 + bz + c = 0$ with $|z_1| = 1$. Let P and Q represent the complex numbers z_1 and z_2 in the Argand plane with $\angle POQ = \theta$, $0° < 180°$ (where O being the origin). Then
(a) $b^2 = ac; \theta = \dfrac{2\pi}{3}$ (b) $\theta = \dfrac{2\pi}{3}; PQ = \sqrt{3}$
(c) $PQ = 2\sqrt{3}; b^2 = ac$ (d) $\theta = \dfrac{\pi}{3}; b^2 = ac$

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Answers Key

Single Correct Answer Type

1. (a) **2.** (d) **3.** (c) **4.** (b) **5.** (c)
6. (b) **7.** (b) **8.** (d) **9.** (a) **10.** (a)
11. (d) **12.** (d)

Multiple Correct Answers Type

13. (b, d) **14.** (a, c) **15.** (a, b)

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DPP 3.4

Properties of Argument

Single Correct Answer Type

1. If $z + \frac{1}{z} = 2\cos 6°$, then $z^{1000} + \frac{1}{z^{1000}} + 1$ is equal to
   (a) 0 (b) 1 (c) −1 (d) 2
2. Let z_1 and z be two complex numbers with α and β as their principal arguments, such that $\alpha + \beta > \pi$. Then principal arg $(z_1 z_2)$ is given by
   (a) $\alpha + \beta + \pi$ (b) $\alpha + \beta - \pi$
   (c) $\alpha + \beta - 2\pi$ (d) $\alpha + \beta$
3. Let arg $(z_k) = \frac{(2k+1)\pi}{n}$ where $k = 1, 2, \ldots, n$. If arg$(z_1, z_2, z_3, \ldots, z_n) = \pi$, then n must be of form $(m \in z)$
   (a) $4m$ (b) $2m - 1$
   (c) $2m$ (d) none of these
4. Suppose two complex numbers $z = a + ib$; $w = c + id$ satisfy the equation $\frac{z+w}{z} = \frac{w}{z+w}$. Then
   (a) both a and c are zeros
   (b) both b and d are zeros
   (c) both b and d must be non zeros
   (d) at least one of b and d is non-zero
5. If $|z| = 1$ and $z \neq \pm 1$, then one of the possible values of arg(z) − arg $(z + 1)$ − arg $(z - 1)$, is
   (a) $-\pi/6$ (b) $\pi/3$ (c) $-\pi/2$ (d) $\pi/4$
6. If arg $(z^{3/8}) = \frac{1}{2}$ arg $(z^2 + \bar{z}z^{1/2})$, then which of the following is not possible?
   (a) $|z| = 1$ (b) $z = \bar{z}$
   (c) arg $(z) = 0$ (d) none of these
7. z_1, z_2 are two distinct points in complex plane such that $2|z_1| = 3|z_2|$ and $z \in C$ be any point $z = \frac{2z_1}{3z_2} + \frac{3z_2}{2z_1}$ such that
   (a) $-1 \leq \text{Re}\, z \leq 1$
   (b) $-2 \leq \text{Re}\, z \leq 2$
   (c) $-3 \leq \text{Re}\, z \leq 3$
   (d) None of these

Multiple Correct Answers Type

8. Let $Z_1 = x_1 + iy_1$, $Z_2 = x_2 + iy_2$ be complex numbers in fourth quadrant of argand plane and $|Z_1| = |Z_2| = 1$, Ref$(Z_1 Z_2) = 0$. The complex numbers $Z_3 = x_1 + ix_2$, $Z_4 = y_1 + iy_2$, $Z_5 = x_1 + iy_2$, $Z_6 = x_2 + iy_1$, will always satisfy
   (a) $|Z_4| = 1$
   (b) arg$(Z_1 Z_4) = -\pi/2$
   (c) $\frac{Z_5}{\cos(\arg Z_1)} + \frac{Z_6}{\sin(\arg Z_1)}$ is purely real
   (d) $Z_5^2 + (\bar{Z}_6)^2$ is purely imaginary
9. If the imaginary part of $\frac{z-3}{e^{i\theta}} + \frac{e^{i\theta}}{z-3}$ is zero, then z can lie on
   (a) a circle with unit radius
   (b) a circle with radius 3 units
   (c) a straight line through the point (3, 0)
   (d) a parabola with the vertex (3, 0)

Answers Key

Single Correct Answer Type

1. (a) **2.** (c) **3.** (b) **4.** (d) **5.** (c)
6. (d) **7.** (b)

Multiple Correct Answers Type

8. (a, b, c, d) **9.** (a, c)

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DPP 3.5

Cube Roots of Units, *n*th Roots of Unity

Single Correct Answer Type

1. If $\alpha, \beta, \gamma \in \{1, \omega, \omega^2\}$ (where ω and ω^2 are imaginary cube roots of unity), then number of triplets (α, β, γ) such that $\left|\frac{a\alpha + b\beta + c\gamma}{a\beta + b\gamma + c\alpha}\right| = 1$, is
   (a) 3 (b) 6 (c) 9 (d) 12

2. The value of $(\sqrt[3]{3} + (3^{5/6})i)^3$ is (where $i = \sqrt{-1}$)
   (a) 24 (b) –24 (c) –22 (d) –21

3. If $\omega \neq 1$ is a cube root of unity and $a + b = 21$, $a^3 + b^3 = 105$, then the value of $(a\omega^2 + b\omega)(a\omega + b\omega^2)$ is be equal to
   (a) 3 (b) 5 (c) 7 (d) 35

4. If $z = \frac{1}{2}(\sqrt{3} - i)$, then the least possible integral value of m such that $(z^{101} + i^{109})^{106} = z^{m+1}$ is
   (a) 11 (b) 7 (c) 8 (d) 9

5. If $y_1 = \max ||z - \omega| - |z - \omega^2||$, where $|z| = 2$ and $y_2 = \max ||z - \omega| - |z - \omega^2||$, where $|z| = \frac{1}{2}$ and ω and ω^2 are complex cube roots of unity, then
   (a) $y_1 = \sqrt{3}$; $y_2 = \sqrt{3}$ (b) $y_1 < \sqrt{3}$; $y_2 = \sqrt{3}$
   (c) $y_1 = \sqrt{3}$; $y_2 < \sqrt{3}$ (d) $y_1 > 3$; $y_2 < \sqrt{3}$

6. Let 1, ω and ω^2 be the cube roots of unity. The least possible degree of a polynomial, with real coefficients, having $2\omega^2$, $3 + 4\omega$, $3 + 4\omega^2$ and $5 - \omega - \omega^2$ as roots is
   (a) 4 (b) 5 (c) 6 (d) 7

7. Number of imaginary complex numbers satisfying the equation, $z^2 = \bar{z} \cdot 2^{1-|z|}$ is
   (a) 0 (b) 1 (c) 2 (d) 3

8. Least positive argument of the 4th root of the complex number $2 + i\sqrt{12}$ is
   (a) $\pi/6$ (b) $5\pi/12$ (c) $7\pi/12$ (d) $11\pi/12$

9. Number of nth roots of unity that are also the roots of the equation $z^2 + az + b = 0$, for some integer a and b is
   (a) 6 (b) 8 (c) 9 (d) 10

10. If z is a complex number satisfying the equation $Z^6 + Z^3 + 1 = 0$. If this equation has a root $re^{i\theta}$ with $90° < \theta < 180°$, then the value of 'θ' is
    (a) 100° (b) 110° (c) 160° (d) 170°

11. Suppose A is a complex number and $n \in N$, such that $A^n = (A + 1)^n = 1$, then the least value of n is
    (a) 3 (b) 6 (c) 9 (d) 12

12. If $z_1, z_2, z_3, \ldots, z_n$ are in G.P with first term as unity such that $z_1 + z_2 + z_3 + \ldots + z_n = 0$. Now if $z_1, z_2, z_3, \ldots, z_n$ represents the vertices of n-polygon, then the distance between incentre and circumcentre of the polygon is
    (a) 0 (b) $|z_1|$ (c) $2|z_1|$ (d) none of these

Multiple Correct Answers Type

13. If α is the fifth root of unity, then
    (a) $|1 + \alpha + \alpha^2 + \alpha^3 + \alpha^4| = 0$
    (b) $|1 + \alpha + \alpha^2 + \alpha^3| = 1$
    (c) $|1 + \alpha + \alpha^2| = 2\cos\frac{\pi}{5}$
    (d) $|1 + \alpha| = 2\cos\frac{\pi}{10}$

14. If z_1, z_2, z_3 are any three roots of the equation $z^6 = (z + 1)^6$, then $\arg\left(\frac{z_1 - z_3}{z_2 - z_3}\right)$ can be equal to
    (a) 0 (b) π
    (c) $\frac{\pi}{4}$ (d) $-\frac{\pi}{4}$

Answers Key

Single Correct Answer Type

1. (c)	2. (b)	3. (b)	4. (d)	5. (c)
6. (b)	7. (c)	8. (b)	9. (b)	10. (c)
11. (b)	12. (a)			

Multiple Correct Answers Type

13. (a, b, c) 14. (a, b)

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DPP 3.6

Geometry Using Complex Numbers (Elementary)

Single Correct Answer Type

1. If $|z - 1 - i| = 1$, then the locus of a point represented by the complex number $5(z - i) - 6$ is
   (a) circle with centre (1, 0) and radius 3
   (b) circle with centre (–1, 0) and radius 5
   (c) line passing through origin
   (d) line passing through (–1, 0)

2. Let $z \in C$ and if $A = \left\{z, \arg(z) = \frac{\pi}{4}\right\}$ and $B = \left\{z, \arg(z - 3 - 3i) = \frac{2\pi}{3}\right\}$. Then $n(A \cap B)$ is equal to
   (a) 1 (b) 2 (c) 3 (d) 0

3. $\theta \in [0, 2\pi]$ and z_1, z_2, z_3 are three complex numbers such that they are collinear and $(1 + |\sin\theta|)z_1 + (|\cos\theta| - 1)z_2 - \sqrt{2}z_3 = 0$. If at least one of the complex numbers z_1, z_2, z_3 is non-zero, then number of possible values of θ is
   (a) Infinite (b) 4 (c) 2 (d) 8

4. Let 'z' be a complex number and 'a' be a real parameter such that $z^2 + az + a^2 = 0$, then which is of the following is not true?
   (a) locus of z is a pair of straight lines
   (b) $|z| = |a|$
   (c) $\arg(z) = \pm\frac{2\pi}{3}$
   (d) none of these

5. Let $z = x + iy$. Then locus of moving point $P(z)$ such that $\frac{1 + \bar{z}}{z} \in R$ is (where $i^2 = -1$)
   (a) union of lines with equations $x = 0$ and $y = -1/2$ but excluding origin.
   (b) union of lines with equations $x = 0$ and $y = 1/2$ but excluding origin.
   (c) union of lines with equations $x = -1/2$ and $y = 0$ but excluding origin.
   (d) union of lines with equations $x = 1/2$ and $y = 0$ but excluding origin.

6. Let $A(z_1)$ and $B(z_2)$ are two distinct non-real complex numbers in the argand plane such that $\frac{z_1}{z_2} + \frac{\bar{z}_1}{\bar{z}_2} = 2$. The value of $|\angle ABO|$ is
   (a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$
   (c) $\frac{\pi}{2}$ (d) none of these

7. Complex numbers z_1 and z_2 satisfy $|z_1| = 2$ and $|z_2| = 3$. If the included angle of their corresponding vectors is $60°$, then the value of $19\left|\frac{z_1 - z_2}{z_1 + z_2}\right|^2$ is
   (a) 5 (b) 6 (c) 7 (d) 8

Matching Column Type

8. z_1, z_2, z_3 are vertices of a triangle. Match the condition in List I with type of triangle in List II.

	List I		List II
(p)	$z_1^2 + z_2^2 + z_3^2 = z_2z_3 + z_3z_1 + z_1z_2$	(1)	right angled but not necessarily iscosceles
(q)	$\text{Re}\left(\frac{z_3 - z_1}{z_3 - z_2}\right) = 0$	(2)	obtuse angled
(r)	$\text{Re}\left(\frac{z_3 - z_1}{z_3 - z_2}\right) < 0$	(3)	isosceles and right angled
(s)	$\frac{z_3 - z_1}{z_3 - z_2} = i$	(4)	equilateral

Codes

	p	q	r	s
(a)	3	2	1	4
(b)	1	2	4	3
(c)	4	1	2	3
(d)	2	1	4	3

Comprehension Type

For Q. 9 to 11

Consider the region R in the Argand plane described by the complex number Z satisfying the inequalities $|Z - 2| \le |Z - 4|$, $|Z - 3| \le |Z + 3|$, $|Z - i| \le |Z - 3i|$, $|Z + i| \le |Z + 3i|$

Answer the following questions:

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9. The maximum value of $|Z|$ for any Z in R is
(a) 5 (b) 3 (c) 1 (d) $\sqrt{13}$

10. Maximum of $|Z_1 - Z_2|$ given that Z_1, Z_2 are any two complex numbers lying in the region R is
(a) 5 (b) 14 (c) $\sqrt{13}$ (d) 12

11. Minimum of $|Z_1 - Z_2|$ given that Z_1, Z_2 are any two complex numbers lying in the region R is
(a) 0 (b) 5 (c) $\sqrt{13}$ (d) 3

Answers Key

Single Correct Answer Type

1. (b) **2.** (d) **3.** (b) **4.** (d) **5.** (c)
6. (c) **7.** (c)

Matching Column Type

8. (c)

Comprehension Type

9. (d) **10.** (a) **11.** (a)

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DPP 3.7

Geometry Using Complex Numbers (Typical)

Single Correct Answer Type

1. Let $A(2, 0)$ and $B(z)$ are two points on the circle $|z| = 2$. $M(z')$ is the point on AB. If fhe point $\overline{z}'$ lies on the median of the triangle OAB where O is origin, then $\arg(z')$ is

(a) $\tan^{-1}\left(\frac{\sqrt{15}}{5}\right)$ (b) $\tan^{-1}(\sqrt{15})$

(c) $\tan^{-1}\left(\frac{5}{\sqrt{15}}\right)$ (d) $\frac{\pi}{2}$

2. If $A(z_1)$, $B(z_2)$, $C(z_3)$ are vertices of a triangle such that $z_3 = \frac{z_2 - iz_1}{1-i}$ and $|z_1| = 3$, $|z_2| = 4$ and $|z_2 + iz_1| = |z_1| + |z_2|$, then area of triangle ABC is

(a) $\frac{5}{2}$ (b) 0

(c) $\frac{25}{2}$ (d) $\frac{25}{4}$

3. Let O, A, B be three collinear points such that $OA \cdot OB = 1$. If O and B represent the complex numbers O and z, then A represents

(a) $\frac{1}{\overline{z}}$ (b) $\frac{1}{z}$

(c) $\overline{z}$ (d) z^2

4. If the tangents at z_1, z_2 on the circle $|z - z_o| = r$ intersect at z_3, then $\frac{(z_3 - z_1)(z_0 - z_2)}{(z_0 - z_1)(z_3 - z_2)}$ equals

(a) 1 (b) -1 (c) i (d) $-i$

5. If z_1, z_2 and z_3 are the vertices of ΔABC, which is not right angled triangle taken in anti-clock wise direction and z_0 is the circumcentre, then $\left(\frac{z_0 - z_1}{z_0 - z_2}\right)\frac{\sin 2A}{\sin 2B} + \left(\frac{z_0 - z_3}{z_0 - z_2}\right)\frac{\sin 2C}{\sin 2B}$ is equal to

(a) 0 (b) 1 (c) -1 (d) 2

6. Let P denotes a complex number $z = r(\cos\theta + i\sin\theta)$ on the Argand's plane, and Q denotes a complex number $\sqrt{2|z|^2}\left(\cos\left(\theta + \frac{\pi}{4}\right) + i\sin\left(\theta + \frac{\pi}{4}\right)\right)$. If '$O$' is the origin, then ΔOPQ is

(a) isosceles but not right angled
(b) right angled but not isosceles
(c) right isosceles
(d) equilateral

Multiple Correct Answers Type

7. Let z_1, z_2, z_3 are the vertices of ΔABC, respectively, such that $\frac{z_3 - z_2}{z_1 - z_2}$ is purely imaginary number. A square on side AC is drawn outwardly. $P(z_4)$ is the centre of square, then

(a) $|z_1 - z_2| = |z_2 - z_4|$

(b) $\arg\left(\frac{z_1 - z_2}{z_4 - z_2}\right) + \arg\left(\frac{z_3 - z_2}{z_4 - z_2}\right) = +\frac{\pi}{2}$

(c) $\arg\left(\frac{z_1 - z_2}{z_4 - z_2}\right) + \arg\left(\frac{z_3 - z_2}{z_4 - z_2}\right) = 0$

(d) z_1, z_2, z_3 and z_4 lie on a circle.

Comprehension Type

For Q. 8 to 10

Let z_1 and z_2 be complex numbers such that $z_1^2 - 4z_2 = 16 + 20i$ and the roots α and β of $x^2 + z_1x + z_2 + m = 0$ for some complex number m satisfies $|\alpha - \beta| = 2\sqrt{7}$.

8. The locus of the complex number m is a curve

(a) straight line (b) circle
(c) ellipse (d) hyperbola

9. The maximum value of $|m|$ is

(a) 14 (b) $2\sqrt{7}$

(c) $7 + \sqrt{41}$ (d) $4 - 2\sqrt{6}$

10. The value of $|m|$, when $\arg(m)$ is maximum

(a) 7 (b) $28 - \sqrt{41}$

(c) $\sqrt{41}$ (d) $2\sqrt{6} - 4$

For Q. 11 to 13

The locus of any point $P(z)$ on argand plane is $\arg\left(\frac{z - 5i}{z + 5i}\right) = \frac{\pi}{4}$.

11. Then the length of the arc described by the locus of $P(z)$ is

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(a) $10\sqrt{2}\pi$ (b) $\frac{15\pi}{\sqrt{2}}$

(c) $\frac{5\pi}{\sqrt{2}}$ (d) $5\sqrt{2}\pi$

12. Total number of integral points inside the region bounded by the locus of P(z) and imaginary axis on the argand plane is

(a) 62 (b) 74

(c) 136 (d) 138

13. Area of the region bounded by the locus of a complex number Z satisfying $\arg\left(\frac{z+5i}{z-5i}\right)=\pm\frac{\pi}{4}$

(a) $75\pi+50$ (b) 75π

(c) $\frac{75\pi}{2}+25$ (d) $\frac{75\pi}{2}$

For Q. 14 and 15

A person walks $2\sqrt{2}$ units away from origin in south west direction (S45°W) to reach A, then walks $\sqrt{2}$ units in south east direction (S45°E) to reach B. From B he travels 4 units horizontally towards east to reach C. Then he travels along a circular path with centre at origin through an angle of $2\pi/3$ in anti-clockwise direction to reach his destination D.

14. Let the complex number Z represents C in argand plane. Then arg(Z) =

(a) $-\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (b) $-\frac{\pi}{4}$ (d) $\frac{\pi}{3}$

15. Position of D in argand plane is (w is an imaginary cube root of unity)

(a) $(3+i)\omega$ (b) $-(1+i)\omega^2$

(c) $3(1-i)\omega$ (d) $(1-3i)\omega$

Answers Key

Single Correct Answer Type

1. (a) **2.** (d) **3.** (a) **4.** (b) **5.** (c)

6. (c)

Multiple Correct Answers Type

7. (c, d)

Comprehension Type

8. (b) **9.** (c) **10.** (d) **11.** (b) **12.** (c)

13. (a) **14.** (c) **15.** (c)

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CHAPTER

4

Progression and Series

DPP 4.1

Arithmetic Progression

Single Correct Answer Type

1. If $3x^2 - 2ax + (a^2 + 2b^2 + 2c^2) = 2(ab + bc)$, then a, b, c can be in
(a) A.P. (b) G.P. (c) H.P. (d) Noneofthese

2. If $x = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots$, $y = \frac{1}{1^2} + \frac{3}{2^2} + \frac{1}{3^2} + \frac{3}{4^2} + \ldots$ and $z = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \ldots$, then
(a) x, y, z are in A.P. (b) $\frac{y}{6}, \frac{x}{3}, \frac{z}{2}$ are in A.P.
(c) $\frac{y}{6}, \frac{x}{3}, \frac{z}{2}$ are in A.P. (d) $6y, 3x, 2z$ are in H.P.

3. For $a, b, c \in R - \{0\}$, let $\frac{a+b}{1-ab}, b, \frac{b+c}{1-bc}$ are in A.P. If α, β are the roots of the quadratic equation $2ac\,x^2 + 2abc\,x + (a + c) = 0$, then the value of $(1 + \alpha)(1 + \beta)$ is
(a) 0 (b) 1 (c) -1 (d) 2

4. If $a_1, a_2, a_3, \ldots, a_{87}, a_{88}, a_{89}$ are the arithmetic means between 1 and 89, then $\sum_{r=1}^{89} \log(\tan(a_r)^\circ)$ is equal to
(a) 0 (b) 1 (c) $\log_2 3$ (d) $\log 5$

5. Let $a_1, a_2, \ldots$ and $b_1, b_2, \ldots$ be arithmetic progression such that $a_1 = 25$, $b_1 = 75$ and $a_{100} + b_{100} = 100$, then the sum of first hundred term of the progression $a_1 + b_1, a_2 + b_2, \ldots$ is equal to
(a) 1000 (b) 100000 (c) 10000 (d) 24000

6. The sum of 25 terms of an A.P., whose all the terms are natural numbers, lies between 1900 and 2000 and its 9^{th} term is 55. Then the first term of the A.P. is
(a) 5 (b) 6 (c) 7 (d) 8

7. If the first, fifth and last terms of an A.P. is l, m, p, respectively, and sum of the A.P. is $\frac{(l+p)(4p+m-5l)}{k(m-l)}$, then k is
(a) 2 (b) 3 (c) 4 (d) 5

8. If $a_1, a_2, a_3, \ldots, a_{15}$ are in A.P. and $a_1 + a_8 + a_{15} = 15$, then $a_2 + a_3 + a_8 + a_{13} + a_{14}$ is equal to
(a) 25 (b) 35 (c) 10 (d) 15

9. If $a_1, a_2, a_3, \ldots$ are in A.P. and $a_i > 0$ for each i, then $\sum_{i=1}^{n} \frac{n}{a_{i+1}^{2/3} + a_{i+1}^{1/3} a_i^{1/3} + a_i^{2/3}}$ is equal to
(a) $\frac{n}{a_n^{2/3} + a_n^{1/3} + a_1^{2/3}}$ (b) $\frac{n+1}{a_n^{2/3} + a_n^{1/3} + a_1^{2/3}}$
(c) $\frac{n-1}{a_n^{2/3} + a_n^{1/3} \cdot a_1^{1/3} + a_1^{2/3}}$ (d) None of these

10. Between the numbers 2 and 20, 8 means are inserted. Then their sum is
(a) 88 (b) 44
(c) 176 (d) None of these

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11. Let $a_1, a_2, a_3, \ldots, a_{4001}$ is an A.P. such that $\frac{1}{a_1a_2}+\frac{1}{a_2a_3}+\ldots+\frac{1}{a_{4000}a_{4001}} = 10$ and $a_2 + a_{400} = 50$.

Then $|a_1 - a_{4001}|$ is equal to

(a) 20 (b) 30
(c) 40 (d) None of these

12. An A.P. consist of even number of terms $2n$ having middle terms equal to 1 and 7 respectively. If n is the maximum value which satisfy $t_1t_{2n} + 713 \geq 0$, then the value of the first term of the series is

(a) 17 (b) −15 (c) 21 (d) −23

13. If the sum of the first 100 terms of an AP is −1 and the sum of even terms lying in first 100 terms is 1, then which of the following is not true?

(a) Common difference of the sequence is $\frac{3}{50}$

(b) First term of the sequence is $\frac{-149}{50}$

(c) 100^{th} term = $\frac{74}{25}$

(d) None of these

14. Given the sequence of numbers $x_1, x_2, x_3, x_4, \ldots, x_{2005}$, which satisfy

$\frac{x_1}{x_1+1}=\frac{x_2}{x_2+3}=\frac{x_3}{x_3+5}=\ldots=\frac{x_{2005}}{x_{2005}+4009}$, the nature of the sequence is

(a) A.P. (b) G.P.
(c) H.P. (d) None of these

Answers Key

Single Correct Answer Type

1. (a) **2.** (b) **3.** (b) **4.** (a) **5.** (c)
6. (c) **7.** (a) **8.** (a) **9.** (c) **10.** (a)
11. (b) **12.** (d) **13.** (d) **14.** (a)

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DPP 4.2

Geometric Progression

Single Correct Answer Type

1. If $b-c$, $bx-cy$, $bx^2-cy^2 (b, c \neq 0)$ are in G.P, then the value of $\left(\frac{bx+cy}{b+c}\right)\left(\frac{bx-cy}{b-c}\right)$ is

(a) x^2 (b) $-x^2$ (c) $2y^2$ (d) $3y^2$

2. If $a_1, a_2, a_3, \ldots$ are in G.P., where $a_i \in C$ (where C stands for set of complex numbers) having r as common ratio such that $\sum_{k=1}^{n} a_{2k-1} = \sum_{k=1}^{n} a_{2k+3} \neq 0$, then the number of possible values of r is

(a) 2 (b) 3 (c) 4 (d) 5

3. If a, b, c are real numbers forming an A.P. and $3+a$, $2+b$, $3+c$ are in G.P., then minimum value of ac is

(a) -4 (b) -6
(c) 3 (d) None of these

4. a, b, c, d are in increasing G.P. If the AM between a and b is 6 and the AM between c and d is 54, then the AM of a and d is

(a) 15 (b) 48 (c) 44 (d) 42

5. The numbers a, b, c are in A.P. and $a+b+c=60$. The numbers $(a-2)$, b, $(c+3)$ are in G.P. Then which of the following is not the possible value of $a^2+b^2+c^2$?

(a) 1208 (b) 1218 (c) 1298 (d) None of these

6. a, b, c are positive integers forming an increasing G.P. and $b-a$ is a perfect cube and $\log_6 a + \log_6 b + \log_6 c = 6$, then $a+b+c =$

(a) 100 (b) 111 (c) 122 (d) 189

7. The first three terms of a geometric sequence are x, y, z and these have the sum equal to 42. If the middle term y is multiplied by 5/4, the numbers x, $\frac{5y}{4}$, z now form an arithmetic sequence. The largest possible value of x is

(a) 6 (b) 12 (c) 24 (d) 20

8. An infinite G.P. has 2nd term x and its sum is 4. Then x belongs to

(a) $(0, 2]$ (b) $(1, 8)$ (c) $(-8, 1]$ (d) none of these

9. In a GP, the ratio of the sum of the first eleven terms to the sum of the last even terms is 1/8 and the ratio of the sum of all the terms without the first nine to the sum of all the terms without the last nine is 2. Then the number of terms in the GP is

(a) 40 (b) 38 (c) 36 (d) 34

Comprehension Type

For Q. 10 and 11

The 1st, 2nd and 3rd terms of an arithmetic series are a, b and a^2 where 'a' is negative. The 1st, 2nd and 3rd terms of a geometric series are a, a^2 and b, respectively.

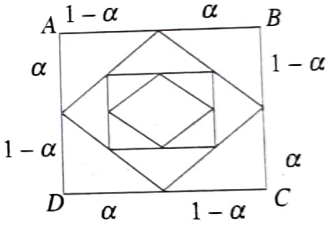
10. The sum of infinite geometric series is

(a) $\frac{-1}{2}$ (b) $\frac{-3}{2}$ (c) $\frac{-1}{3}$ (d) none of these

11. The sum of the 40 terms of the arithmetic series is

(a) $\frac{545}{2}$ (b) 220 (c) 250 (d) $\frac{575}{2}$

For Q. 12 to 14



Let $ABCD$ is a unit square and each side of the square is divided in the ratio $\alpha : (1-\alpha)$ $(0 < \alpha < 1)$. These points are connected to obtain another square. The sides of new square are divided in the ratio $\alpha : (1-\alpha)$ and points are joined to obtain another square. The process is continued indefinitely. Let a_n denote the length of side and A_n the area of the n^{th} square

12. If $\alpha = \frac{1}{3}$, then the least value of n for which $A_n < \frac{1}{10}$ is

(a) 4 (b) 5 (c) 6 (d) 7

13. The value of α for which $\sum_{n=1}^{\infty} A_n = \frac{8}{3}$ is/are

(a) $\frac{1}{3}, \frac{2}{3}$ (b) $\frac{1}{4}, \frac{3}{4}$ (c) $\frac{1}{5}, \frac{4}{5}$ (d) $\frac{1}{2}$

14. The value of α for which side of n^{th} square equal to the diagonal of $(n+1)^{th}$ square is

(a) $\frac{1}{3}$ (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) $\frac{1}{\sqrt{2}}$

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Answers Key

Single Correct Answer Type

1. (a) **2.** (c) **3.** (b) **4.** (d) **5.** (b)
6. (d) **7.** (c) **8.** (c) **9.** (b)

Comprehension Type

10. (c) **11.** (a) **12.** (b) **13.** (b) **14.** (c)

Harmonic Progression and Relation Between Means

Single Correct Answer Type

1. The number of ordered pairs (x, y), where $x, y \in N$ for which $4, x, y$ are in H.P. , is equal to
(a) 1 (b) 2 (c) 3 (d) 4

2. If $a+c, a+b, b+c$ are in G.P. and a, c, b are in H.P. where $a, b, c > 0$, then the value of $\frac{a+b}{c}$ is
(a) 3 (b) 2 (c) $\frac{3}{2}$ (d) 4

3. If a, b, c are in H.P., b, c, d are in G.P. and c, d, e are in A.P., then the value of e is
(a) $\frac{ab^2}{(2a-b)^2}$ (b) $\frac{a^2b}{(2a-b)^2}$
(c) $\frac{a^2b^2}{(2a-b)^2}$ (d) None of these

4. If $x>1, y>1, z>1$ are in G.P, then $\log_{ex} e, \log_{ey} e, \log_{ez} e$ are in
(a) A.P. (b) H.P.
(c) G.P. (d) none of these

5. If x, y, z are in G.P. $(x, y, z > 1)$, then $\frac{1}{2x+\log_e x}, \frac{1}{4x+\log_e y}, \frac{1}{6x+\log_e z}$ are in
(a) A.P. (b) G.P.
(c) H.P. (d) none of these

6. The arithmetic mean of two positive numbers is 6 and their geometric mean G and harmonic mean H satisfy the relation $G^2+3H=48$. Then the product of the two numbers is
(a) 24 (b) 32 (c) 48 (d) 54

7. If x, y, z be three numbers in G.P. such that 4 is the A.M. between x and y and 9 is the H.M. between y and z, then y is
(a) 4 (b) 6 (c) 8 (d) 12

8. If harmonic mean of $\frac{1}{2}, \frac{1}{2^2}, \frac{1}{2^3}, \ldots, \frac{1}{2^{10}}$ is $\frac{\lambda}{2^{10}-1}$, then $\lambda =$
(a) 10.2^{10} (b) 5 (c) 5.2^{10} (d) 10

9. An aeroplane flys around squares whose all sides are of length 100 miles. If the aeroplane covers at a speed of 100 mph the first side, 200 mph the second side 300 mph the third side and 400 mph the fourth side. The average speed of aeroplane around the square is
(a) 190 mph (b) 195 mph
(c) 192 mph (d) 200 mph

Multiple Correct Answers Type

10. Let a, x, b be in A.P; a, y, b be in G.P and a, z, b be in H.P. If $x = y + 2$ and $a = 5z$, then
(a) $y^2 = xz$ (b) $x > y > z$
(c) $a = 9, b = 1$ (d) $a = 1/4, b = 9/4$

11. If $A_1, A_2, A_3; G_1, G_2, G_3$; and H_1, H_2, H_3 are the three arithmetic, geometric and harmonic means between two positive numbers a and b $(a > b)$, then which of the following is/are ture?
(a) $2G_1G_3 = H_2(A_1+A_3)$ (b) $A_2H_2 = G_2^2$
(c) $A_2G_2 = H_2^2$ (d) $2G_1A_1 = H_1(A_1+A_3)$

12. Given that α, γ are roots of the equation $Ax^2 - 4x + 1 = 0$ and β, δ are roots of the equation $Bx^2 - 6x + 1 = 0$. If α, β, γ and δ are in H.P., then
(a) $A = 5$ (b) $A = 3$
(c) $B = 8$ (d) $B = -8$

13. If $\frac{1}{a}+\frac{1}{c} = \frac{1}{2b-a}+\frac{1}{2b-c}$, then
(a) a, b, c are in A.P. (b) $a, \frac{b}{2}, c$ are in A.P.
(c) $a, \frac{b}{2}, c$ are in H.P. (d) $a, 2b, c$ are in H.P.

Answers Key

Single Correct Answer Type

1. (c) **2.** (b) **3.** (a) **4.** (b) **5.** (c)
6. (b) **7.** (b) **8.** (b) **9.** (c)

Multiple Correct Answers Type

10. (a, b, c) **11.** (a, b)
12. (b, c) **13.** (a, d)

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DPP 4.4

Sigma

Single Correct Answer Type

1. The sum of the series $1+\frac{9}{4}+\frac{36}{9}+\frac{100}{16}+\ldots$ infinite terms is
   (a) 446 (b) 746
   (c) 546 (d) 846

2. The sum $2\times5+5\times9+8\times13+\ldots$ 10 terms is
   (a) 4500 (b) 4555
   (c) 5454 (d) None of these

3. The sum of n terms of series $ab+(a+1)(b+1)+(a+2)(b+2)+\ldots+(a+(n-1))(b+n-1))$ if $ab=\frac{1}{6}$ and $(a+b)=\frac{1}{3}$ is
   (a) $\frac{n}{6}(1-2n)^2$ (b) $\frac{n}{6}(1+n-2n^2)$
   (c) $\frac{n}{6}(1-2n+2n^2)$ (d) none of these

4. $\sum_{i=1}^{\infty}\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\frac{1}{a^{i+j+k}}$ is equal to (where $|a|>1$)
   (a) $(a-1)^{-3}$ (b) $\frac{3}{a-1}$
   (c) $\frac{3}{a^3-1}$ (d) None of these

5. The coefficient of x^{1274} in the expansion of $(x+1)(x-2)^2(x+3)^3(x-4)^4\ldots(x+49)^{49}(x-50)^{50}$ is
   (a) 1275 (b) -1275
   (c) $-\sum_{i=1}^{50}i^2$ (d) $-\sum_{i=1}^{50}i^2$

6. If the positive integers are written in a triangular array as shown below,

   1
   2 3
   4 5 6
   7 8 9 10
   .....
   .....

   then the row in which the number 2010 will be, is
   (a) 65 (b) 61 (c) 63 (d) 65

7. The value of $\sum_{i=1}^{n}\sum_{j=1}^{i}\sum_{k=1}^{j}1=220$, then the value of n equals
   (a) 11 (b) 12 (c) 10 (d) 9

8. The sum $\sum_{k=1}^{10}\sum_{\substack{j=1\\ i\neq j\neq k}}^{10}\sum_{i=1}^{10}1$ is equal to
   (a) 240 (b) 720 (c) 540 (d) 1080

9. The sum $\sum_{k=1}^{10}\sum_{\substack{j=1\\ i<j<k}}^{10}\sum_{i=1}^{10}1$ is equal to
   (a) 120 (b) 240 (c) 360 (d) 720

Comprehension Type

For Q. 10 and 11

Let $f(n)$ denote the n^{th} terms of the sequence 3, 6, 11, 18, 27, ... and $g(n)$ denote the n^{th} term of the sequence 3, 7, 13, 21, ... Let $F(n)$ and $G(n)$ denote the sum of n terms of the above sequences, respectively. Now answer the following:

10. $\lim_{n\to\infty}\frac{f(n)}{g(n)}=$
    (a) 0 (b) 1 (c) 2 (d) ∞

11. $\lim_{n\to\infty}\frac{F(n)}{G(n)}=$
    (a) 2 (b) 1 (c) 0 (d) ∞

Answers Key

Single Correct Answer Type

1. (a) 2. (b) 3. (c) 4. (a) 5. (b)
6. (c) 7. (c) 8. (b) 9. (a)

Comprehension Type

10. (b) 11. (b)

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Miscellaneous Series

Single Correct Answer Type

1. If the sum to infinity of the series, $1 + 4x + 7x^2 + 10x^3 + \ldots$ is $\frac{35}{16}$, where $|x| < 1$, then 'x' equals to

(a) 19/7 (b) 1/5 (c) 1/4 (d) None of these

2. The value of $\sum_{n=1}^{\infty}(-1)^{n+1}\left(\frac{n}{5^n}\right)$ equals

(a) $\frac{5}{12}$ (b) $\frac{5}{24}$ (c) $\frac{5}{36}$ (d) $\frac{5}{16}$

3. Find the sum of the infinite series $\frac{1}{9}+\frac{1}{18}+\frac{1}{30}+\frac{1}{45}+\frac{1}{63}+\ldots$

(a) $\frac{1}{3}$ (b) $\frac{1}{4}$ (c) $\frac{1}{5}$ (d) $\frac{2}{3}$

4. If $\sum_{r=1}^{r=n}\frac{r^4+r^2+1}{r^4+r}=\frac{675}{26}$, then n is equal to

(a) 10 (b) 15 (c) 25 (d) 30

5. The sequence $\{x_k\}$ is defined by $x_{k+1} = x_k^2 + x_k$ and $x_1 = \frac{1}{2}$. Then $\left[\frac{1}{x_1+1}+\frac{1}{x_2+1}+\ldots+\frac{1}{x_{100}+1}\right]$ (where [.] denotes the greatest integer function) is equal to

(a) 0 (b) 2 (c) 4 (d) 1

6. The absolute value of the sum of first 20 terms of series, if $S_n=\frac{n+1}{2}$ and $\frac{T_{n-1}}{T_n}=\frac{1}{n^2}-1$, where n is odd, given S_n and T_n denotes sum of first n terms and n^{th} term of the series

(a) 340 (b) 430 (d) 230 (d) 320

7. If $S_n = (1^2 - 1 + 1)(1!) + (2^2 - 2 + 1)(2!) + \ldots + (n^2 - n + 1)(n!)$, then $S_{50} =$

(a) 52! (b) $1 + 49 \times 51!$

(c) $52! - 1$ (d) $50 \times 51! - 1$

8. If $S_n = \frac{1.2}{3!}+\frac{2.2^2}{4!}+\frac{3.2^2}{5!}+\ldots+$ up to n terms, then sum of infinite terms is

(a) $\frac{4}{\pi}$ (b) $\frac{3}{e}$ (c) $\frac{\pi}{r}$ (d) 1

9. There is a certain sequence of positive real numbers. Beginning from the third term, each term of the sequence is the sum of all the previous terms. The seventh term is equal to 1000 and the first term is equal to 1. The second term of this sequence is equal to

(a) 246 (b) $\frac{123}{2}$ (c) $\frac{123}{4}$ (d) 124

10. The sequence $\{x_1, x_2, \ldots, x_{50}\}$ has the property that for each k, x_k is k less than the sum of other 49 numbers. The value of $96x_{20}$ is

(a) 300 (b) 315 (c) 1024 (d) 0

11. Let $a_0 = 0$ and $a_n = 3a_{n-1} + 1$ for $n \geq 1$. Then the remainder obtained dividing a_{2010} by 11 is

(a) 0 (b) 7 (c) 3 (d) 4

12. Suppose $a_1, a_2, a_3, \ldots, a_{2012}$ are integers arranged on a circle. Each number is equal to the average of its two adjacent numbers. If the sum of all even indexed numbers is 3018, what is the sum of all numbers?

(a) 0 (b) 9054

(c) 12072 (d) 6036

13. The sum of the series $\frac{9}{5^2\cdot 2\cdot 1}+\frac{13}{5^3\cdot 3\cdot 2}+\frac{17}{5^4\cdot 4\cdot 3}+\ldots$ upto infinity

(a) 1 (b) $\frac{9}{5}$ (c) $\frac{1}{5}$ (d) $\frac{2}{5}$

Answers Key

Single Correct Answer Type

1. (b) **2.** (c) **3.** (a) **4.** (c) **5.** (d)
6. (b) **7.** (b) **8.** (d) **9.** (b) **10.** (b)
11. (a) **12.** (d) **13.** (c)

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CHAPTER

5

Inequalities Involving Means

DPP 5.1

Inequalities (Level 1)

Single Correct Answer Type

1. If a, b, c be three positive numbers in A.P. and $E = \frac{a+8b}{2b-a} + \frac{8b+c}{2b-c}$, then a value of E can be

(a) 16 (b) 15 (c) 17 (d) 21

2. Let $ab = 1$, then the minimum value of $\frac{1}{a^4} + \frac{1}{4b^4}$ is

(a) 1 (b) 2 (c) 1/4 (d) 1/2

3. If $x > 0, y > 0, z > 0$, the least value of $x^{\log_e y - \log_e z} + y^{\log_e z - \log_e x} + z^{\log_e x - \log_e y}$ is

(a) 3 (b) 1 (c) 5 (d) 6

4. Let $p, q, r \in R^+$ and $27pqr \geq (p+q+r)^3$ and $3p + 4q + 5r = 12$. Then the value of $8p + 4q - 7r =$

(a) 2 (b) 3 (c) 4 (d) 5

5. Minimum value of $f(x) = \cos^2 x + \frac{\sec x}{4}, x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ is

(a) 3/2 (b) 3/4 (c) 3/8 (d) none of these

6. Let $x, y, z \in R^+$ and $2xy + 3yz + 4xz = 18$. If α, β and γ be the values of x, y and z, respectively, for which xyz attains its maximum value, then the value of $2\alpha + \beta + \gamma =$

(a) 4 (b) 6 (c) 8 (d) 12

7. The minimum value of $\frac{x^4 + y^4 + z^2}{xyz}$ for positive real numbers x, y, z is

(a) $\sqrt{2}$ (b) $2\sqrt{2}$ (c) $4\sqrt{2}$ (d) $8\sqrt{2}$

8. If $n_1, n_2, n_3, \ldots, n_{100}$ are positive real numbers such that $n_1 + n_2 + n_3 + \ldots + n_{100} = 20$ and $k = n_1(n_2 + n_3 + n_4)(n_5 + n_6 + \ldots + n_9)(n_{10} + \ldots + n_{16}) \ldots (\ldots + n_{100})$, then k belongs to

(a) (0, 100] (b) (0, 128]
(c) [0, 144] (d) (0, 1024]

9. If a, b, c are the sides of a triangle, then the least value of $\frac{a}{c+a-b} + \frac{b}{a+b-c} + \frac{c}{b+c-a}$ is

(a) 1/3 (b) 1 (c) 3 (d) 6

10. If $x > 0$, $\frac{x^n}{1 + x + x^2 + \cdots + x^{2n}}$ is

(a) $\leq \frac{1}{2n+1}$ (b) $< \frac{2}{2n+1}$

(c) $\geq \frac{1}{2n+1}$ (d) $> \frac{2}{2n+1}$

Answers Key

Single Correct Answer Type

1. (d) **2.** (a) **3.** (a) **4.** (c) **5.** (b)
6. (b) **7.** (b) **8.** (d) **9.** (b) **10.** (a)

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DPP 5.2

Inequalities (Level 2)

Single Correct Answer Type

1. If positive quantities a, b, c are in H.P., then which of the following is not true?

(a) $b < \frac{a+c}{2}$ (b) $\frac{1}{a-b} - \frac{1}{b-c} < 0$

(c) $ac > b^2$ (d) none of these

2. Given that x, y, z are positive real numbers such that $xyz = 32$, the minimum value of $\sqrt{(x+2y)^2 + 2z^2 - 15}$ is equal to

(a) 6 (b) 8 (c) 9 (d) 12

3. If x, y, z are positive real numbers such that $x^2 + y^2 + z^2 = 7$ and $xy + yz + xz = 4$ then the minimum value of xy is

(a) 1 (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) $\frac{1}{8}$

4. If x lies between –5 and 11, then the greatest value of $(11-x)^3(x+5)^5$ is

(a) $6^5 \cdot 10^3$ (b) $6^3 \cdot 10^3$

(c) $6^3 \cdot 10^4$ (d) $6^3 \cdot 10^5$

5. The least integral value of

$$f(x) = \frac{(x-1)^7 + 3(x-1)^6 + (x-1)^5 + 1}{(x-1)^5}, \forall x > 1 \text{ is}$$

(a) 8 (b) 6 (c) 12 (d) 18

6. If a, b, c are positive real numbers and $2a + b + 3c = 1$, then the maximum value of $a^4b^2c^2$ is equal to

(a) $\frac{1}{3.4^8}$ (b) $\frac{1}{9.4^7}$

(c) $\frac{1}{9.4^8}$ (d) $\frac{1}{27.4^8}$

7. If x, y, z be three positive numbers such that xyz^2 has the greatest value $\frac{1}{64}$, then the value of $\frac{1}{x} + \frac{1}{y} + \frac{1}{z}$ is

(a) 6 (b) 8 (c) 10 (d) 12

8. If x_1, x_2 and x_3 are the positive roots of the equation $x^3 - 6x^2 + 3px - 2p = 0, p \in R$, then the value of

$$\sin^{-1}\left(\frac{1}{x_1} + \frac{1}{x_2}\right) + \cos^{-1}\left(\frac{1}{x_2} + \frac{1}{x_3}\right) - \tan^{-1}\left(\frac{1}{x_3} + \frac{1}{x_1}\right) \text{ is}$$

equal to

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$ (c) $\frac{3\pi}{4}$ (d) π

Comprehension Type

For Q. 9 and 10

If a, b, x, y are real number and $x, y > 0$, then $\frac{a^2}{x} + \frac{b^2}{y} \geq \frac{(a+b)^2}{x+y}$

as on solving it we have $(ay - bx)^2 \geq 0$.

Similarly, we can extend the inequality to three pairs of numbers, i.e.,

$$\frac{a^2}{x} + \frac{b^2}{y} + \frac{c^2}{z} \geq \frac{(a+b+c)^2}{x+y+z}$$

Now use this result to solve the following questions.

9. The value of $\frac{a^2+b^2}{a+b} + \frac{b^2+c^2}{b+c} + \frac{a^2+c^2}{a+c}$ is

(a) $\geq (a+b+c)$ (b) $\geq \frac{1}{2}(a+b+c)$

(c) $\frac{3}{2} \geq (a+b+c)$ (d) None of these

10. If $abc = 1$, then the minimum value of

$$\frac{1}{a^3(b+c)} + \frac{1}{b^3(a+c)} + \frac{1}{c^3(a+b)} \text{ is}$$

(a) 3 (b) 3/2 (c) 6 (d) 9

Answers Key

Single Correct Answer Type

1. (d) **2.** (c) **3.** (c) **4.** (d) **5.** (b)
6. (c) **7.** (c) **8.** (a)

Comprehension Type

9. (a) **10.** (b)

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6 Permutation and Combination

DPP 6.1

Counting Principle

Single Correct Answer Type

1. 116 people participated in a knockout tennis tournament. The players are paired up in the first round, the winners of the first round are paired up in the second round, and so on till the final is played between two players. If after any round, there is odd number of players, one player is given a bye, i.e. he skips that round and plays the next round with the winners. The total number of matches played in the tournament is
   (a) 115 (b) 53 (c) 232 (d) 116
2. The number of three-digit numbers having only two consecutive digits identical is
   (a) 153 (b) 162 (c) 180 (d) 161
3. How many six-digit numbers are there in which no digit is repeated, even digits appear at even places, odd digits appear at odd places and the number is divisible by 4?
   (a) 3600 (b) 2700 (c) 2160 (d) 1440
4. The number of ordered pairs (m, n) where $m, n \in \{1, 2, 3, \ldots, 50\}$, such that $6^m + 9^n$ is a multiple of 5 is
   (a) 1250 (b) 2500 (c) 625 (d) 500
5. There are 10 different books in a shelf. The number of ways in which three books can be selected so that exactly two of them are consecutive is.
   (a) 60 (b) 54 (c) 56 (d) 36
6. The number of ways of arranging 6 players to throw the cricket ball so that the oldest player may not throw first is
   (a) 120 (b) 600 (c) 720 (d) 7156
7. Number of four-digit positive integers if the product of their digits is divisible by 3 is.
   (a) 2700 (b) 5464 (c) 6628 (d) 7704
8. The number of five-digit numbers which are divisible by 3 that can be formed by using the digits 1, 2, 3, 4, 5, 6, 7, 8 and 9, when repetition of digits is allowed, is
   (a) 3^9 (b) 4.3^8 (c) 5.3^8 (d) 7.3^8
9. If N is the number of positive integral solutions of $x_1x_2x_3x_4 = 770$, then $N =$
   (a) 256 (b) 729 (c) 900 (d) 770
10. I have tiled my square bathroom wall with congruent square tiles. All the tiles are red, except those along the two diagonals, which are all blue. If I used 121 blue tiles, then the number of red tiles I used are
    (a) 900 (b) 1800 (c) 3600 (d) 7200
11. The number of ordered pairs of positive integers (m, n) satisfying $m \le 2n \le 60$, $n \le 2m \le 60$ is
    (a) 240 (b) 480
    (c) 960 (d) none of these
12. Number of ways in which 6 distinct objects can be kept into two identical boxes so that no box remains empty is
    (a) 31 (b) 32 (c) 63 (d) 64
13. The number of four-digit numbers that can be formed by using the digits 1, 2, 3, 4, 5, 6, 7, 8 and 9 such that the least digit used is 4, when repetition of digits is allowed, is
    (a) 617 (b) 671 (c) 716 (d) 761
14. A fair coin is tossed n times. Let a_n denotes the number of cases in which no two heads occur consecutively. Then which of the following is not true?
    (a) $a_1 = 2$ (b) $a_2 = 3$
    (c) $a_5 = 13$ (d) $a_8 = 55$

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Answers Key

Single Correct Answer Type

1. (a) **2.** (b) **3.** (d) **4.** (a) **5.** (c) **6.** (b) **7.** (d) **8.** (a) **9.** (a) **10.** (a)
11. (b) **12.** (a) **13.** (b) **14.** (c)

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DPP 6.2

Permutation

Single Correct Answer Type

1. Five boys and three girls are sitting in a row of 8 seats. Number of ways in which they can be seated so that not all the girls sit side by side is
   (a) 36000 (b) 9080
   (c) 3960 (d) 11600
2. Number of words that can be made with the letters of the word "GENIUS" if each word neither begins with G nor ends in S is
   (a) 24 (b) 240 (c) 480 (d) 504
3. The number of ways in which the letters of the word PESSIMISTIC can be arranged so that no two S's are together, no two I's are together and letters S and I are never together is
   (a) 8640 (b) 4800
   (c) 2400 (d) 5480
4. The number of different words that can be formed using all the letters of the word 'SHASHANK' such that in any word the vowels are separated by atleast two consonants, is
   (a) 2700 (b) 1800 (c) 900 (d) 600
5. The number of ways in which six boys and six girls can be seated at a round table so that no two girls sit together and two particular girls do not sit next to a particular boy is
   (a) 6! 4! (b) 2.5! 4!
   (c) 2.6! 4! (d) 5! 4!
6. Six cards are drawn one by one from a set of unlimited number of cards, each card is marked with numbers $-1, 0$ or 1. Number of different ways in which they can be drawn if the sum of the numbers shown by them vanishes is
   (a) 111 (b) 121 (c) 141 (d) none of these
7. The number of positive six-digit integers which are divisible by 9 and four of its digits are 1, 0, 0, 5 is
   (a) 60 (b) 120 (c) 180 (d) 210
8. Number of nine-lettered word that can be formed using all the letters of the word 'MEENANSHU' if alike letters are never adjacent is
   (a) $12 \times 6!$ (b) $11 \cdot 7!$
   (c) $13 \cdot 6!$ (d) $12 \cdot 11 \cdot 6!$
9. Number of ways in which the letters of the word 'ABBCABBC' can be arranged such that the word ABBC does not appear is any word, is
   (a) 256 (b) 391 (c) 361 (d) 498
10. The number of 4 digit natural numbers such that the product of their digits is 12 is
    (a) 24 (b) 36 (c) 42 (d) 48
11. A class has three teachers, Mr. X, Ms. Y and Mrs. Z and six students A, B, C, D, E, F. Number of ways in which they can be seated in a line of 9 chairs, if between any two teachers there are exactly two students is
    (a) $18 \times 6!$ (b) $12 \times 6!$
    (c) $24 \times 6!$ (d) $6 \times 6!$

Answers Key

Single Correct Answer Type

1. (a) 2. (d) 3. (c) 4. (a) 5. (c)
6. (c) 7. (c) 8. (b) 9. (c) 10. (b)
11. (a)

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DPP 6.3

Combination Level 1

Single Correct Answer Type

1. The number of words that can be formed using all the letters of the word REGULATIONS such that G must come after R, L must come after A, and S must come after N are
(a) 11!/8 (b) 11! (c) $^{11}P_6$ (d) None of these

2. The number of different permutations of all the letters of the word 'PERMUTATION' such that any two consecutive letters in the arrangement are neither both vowels nor both identical is
(a) $63 \times 6! \times 5!$ (b) $57 \times 5! \times 5!$
(c) $33 \times 6! \times 5!$ (d) $7 \times 7! \times 5!$

3. A guard of 12 men is formed from a group of 'n' soldiers. It is found that 2 particular soldiers A and B are 3 times as often together on guard as 3 particulars soldiers C, D and E. Then n is equal to
(a) 28 (b) 27 (c) 32 (d) 36

4. There are 10 stations on a circular path. A train has to stop at 3 stations such that no two stations are adjacent. The number of such selections must be
(a) 50 (b) 60 (c) 70 (d) 80

5. The number of ways of arranging 15 students $A_1, A_2, \ldots, A_{15}$ in a row such that neither A_2 nor A_3 be seated before A_1 is
(a) $\frac{2 \times 15!}{3!}$ (b) $\frac{15!}{3!}$
(c) 2!15! (d) None of these

6. There are 15 different apples and 10 different pears. How many ways are there for Jack to pick an apple or a pear and then Jill to pick an apple and a pear?
(a) 23×150 (b) 33×150
(c) 43×150 (d) 53×150

7. There are 12 pairs of shoes in a box. Then the possible number of ways of picking 7 shoes so that there are exactly two pairs of shoes are
(a) 63360 (b) 63300
(c) 63260 (d) 63060

8. There are two sets of parallel lines, their equations being $x \cos \alpha + y \sin \alpha = p$ and $x \sin \alpha - y \cos \alpha = p$; $p = 1, 2, 3, \ldots, n$ and $\alpha \in (0, \pi/2)$. If the number of rectangles formed by these two sets of lines is 225, then the value of n is equals to
(a) 4 (b) 5 (c) 6 (d) 7

9. The number of rectangles that can be obtained by joining four of the 12 vertices of a 12-sided regular polygon is
(a) 66 (b) 30 (c) 24 (d) 15

10. The interior angles of a regular polygon measure 150° each. The number of diagonals of the polygon is
(a) 35 (b) 44 (c) 54 (d) 78

11. Number of ways in which 7 green bottles and 8 blue bottles can be arranged in a row if exactly 1 pair of green bottles is side by side is (assume all bottles to be alike except for the colour).
(a) 84 (b) 360 (c) 504 (d) none of these

12. Which of the following is not the number of ways of selecting n objects from $2n$ objects of which n objects are identical
(a) 2^n
(b) $(^{2n+1}C_0 + {}^{2n+1}C_1 + \ldots + {}^{2n+1}C_n)^{1/2}$
(c) the number of possible subsets $\{a_1, a_2, \ldots, a_n\}$
(d) None of these

13. A seven-digit number is in the form of $abcdefg$ ($g, f, e, \ldots$ etc. are digits at units, tens, hundred place ....etc), where $a < b < c < d > e > f > g$ and a, b, c, e, f, g are different digits. The number of such numbers is
(a) 1980 (b) 1116 (c) 1560 (d) 1476

14. The number of all six-digit numbers using digits 1, 2, 3, 4, 5 such that any digit that appears in the number appears at least twice is
(a) 1500 (b) 1850 (c) 1405 (d) 1205

15. All the five-digit numbers $N = abcde$ having property $a < b < c < d < e$ are arranged in the increasing order of their magnitude. The 97th number in the list does not contain the digit
(a) 4 (b) 5 (c) 7 (d) 8

Answers Key

Single Correct Answer Type

1. (a) **2.** (b) **3.** (c) **4.** (a) **5.** (a)
6. (a) **7.** (a) **8.** (c) **9.** (d) **10.** (c)
11. (c) **12.** (d) **13.** (c) **14.** (c) **15.** (b)

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DPP 6.4

Combination Level 2

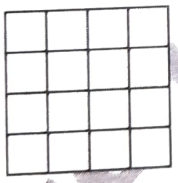
Single Correct Answer Type

1. The number of n digit numbers formed by using digits $\{1, 2, 3\}$ such that if 1 appears, it appears even number of times.
   (a) $2^n + 1$ (b) $\frac{1}{2}(3^n + 1)$
   (c) $\frac{1}{2}(3^n - 1)$ (d) $\frac{1}{2}(2^n - 1)$

2. A, B, C, D develop 18 items. Five items jointly by A and C, four items by A and D, four items by B and C and five items by B and D. The number of ways of selecting eight items out of 18 so that the selected ones belong equally to A, B, C, D is
   (a) 5226 (b) 5626
   (c) 4418 (d) 4936

3. The number of pairs of diagonals of a regular polygon of 10 sides that are parallel are
   (a) 45 (b) 56 (c) 22 (d) 64

4. Four letters, two 'a' and two 'b' are filled into 16 cells of a matrix as given. It is required that each cell contains atmost one letter and each row or column cannot contain same letters. Then the number of ways the matrix can be filled is



   (a) 3600 (b) 5200
   (c) 3960 (d) 4120

5. The number of increasing function from $f: A \to B$ where $A \in \{a_1, a_2, a_3, a_4, a_5, a_6\}$, $B \in \{1, 2, 3, \ldots, 9\}$ such that $a_{i+1} > a_i \ \forall\, i \in N$ and $a_i \neq i$ is
   (a) 30 (b) 28 (c) 24 (d) 42

6. How many ordered pairs of (m, n) integers satisfy $\frac{m}{12} = \frac{12}{n}$?
   (a) 30 (b) 15 (c) 12 (d) 10

7. Product of all the even divisors of $N = 1000$ is
   (a) $2^{20} \cdot 5^{20}$ (b) $2^{24} \cdot 5^{24}$
   (c) $64 \cdot 10^{18}$ (d) None of these

8. How many combinations can be made up of 3 hens, 4 ducks and 2 geese so that each combination has hens, ducks and geese?
   (a) 305 (b) 315 (c) 320 (d) 325

9. A positive integer n is of the form $n = 2^{\alpha} 3^{\beta}$, where $\alpha \geq 1$, $\beta \geq 1$. If n has 12 positive divisors and $2n$ has 15 positive divisors, then the number of positive divisors of $3n$ is
   (a) 15 (b) 16 (c) 18 (d) 20

10. Number of permutations of 1, 2, 3, 4, 5, 6, 7, 8, and 9 taken all at a time are such that digit 1 appearing somewhere to the left of 2 and digit 3 appearing to the left of 4 and digit 5 somewhere to the left of 6, is
    (e.g. 815723946 would be one such permutation)
    (a) 9.7! (b) 8! (c) 5!.4! (d) 8!.4!

11. The number of arrangements of all digits of 12345 such that at least 3 digits will not come in its position is
    (a) 89 (b) 109 (c) 78 (d) 57

Multiple Correct Answers Type

12. A contest consists of ranking 10 songs of which 6 are Indian classic and 4 are western songs. Number of ways of ranking so that (mention correct statements)
    (a) There are exactly 3 Indian classic songs in top 5 is $(5!)^3$.
    (b) Top rank goes to Indian classic song is $6(9!)$.
    (c) The ranks of all western songs are consecutive is $4!7!$.
    (d) The 6 Indian classic songs are in a specified order is ${}^{10}P_4$.

13. $P = n(n^2 - 1)(n^2 - 4)(n^2 - 9) \ldots (n^2 - 100)$ is always divisible by; $(n \in I)$
    (a) 2! 3! 4! 5! 6! (b) $(5!)^4$
    (c) $(10!)^2$ (d) 10! 11!

Comprehension Type

For Q. 14 and 15

Given are six 0's, five 1's and four 2's. Consider all possible permutations of all these numbers. [A permutation can have its leading digit 0].

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14. How many permutations have the first 0 preceding the first 1?

(a) ${}^{15}C_4 \times {}^{10}C_5$ (b) ${}^{15}C_5 \times {}^{10}C_4$

(c) ${}^{15}C_6 \times {}^{10}C_5$ (d) ${}^{15}C_5 \times {}^{10}C_5$

15. In how many permutations does the first 0 precede the f
1 and the first 1 precede first 2.

(a) ${}^{14}C_5 \times {}^{8}C_6$ (b) ${}^{14}C_5 \times {}^{8}C_4$

(c) ${}^{14}C_6 \times {}^{8}C_4$ (d) ${}^{14}C_6 \times {}^{8}C_6$

Answers Key

Single Correct Answer Type

1. (b) **2.** (b) **3.** (a) **4.** (c) **5.** (b)
6. (a) **7.** (b) **8.** (b) **9.** (b) **10.** (a)
11. (b)

Multiple Correct Answers Type

12. (a, b, c, d) **13.** (a, b, c, d)

Comprehension Type

14. (a) **15.** (b)

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DPP 6.5

Division and Distribution and Miscellaneous

Single Correct Answer Type

1. Sixteen players $S_1, S_2, S_3, \ldots, S_{16}$ play in a tournament. Number of ways in which they can be grouped into eight pairs so that S_1 and S_2 are in different groups, is equal to
   (a) $\frac{(14)!}{2^6 \cdot 6!}$ (b) $\frac{(15)!}{2^7 \cdot 7!}$
   (c) $\frac{(14)!}{2^7 \cdot 6!}$ (d) $\frac{(14)!}{2^6 \cdot 7!}$
2. The number of homogenous products of degree 3 from 4 variables is equal to
   (a) 20 (b) 16 (c) 12 (d) 4
3. The number of ways of distributing 3 identical physics books and 3 identical mathematics books among three students such that each student gets at least one book is
   (a) 45 (b) 55 (c) 64 (d) 72
4. Four different movies are running in a town. Ten students go to watch these four movies. The number of ways in which every movie is watched by atleast one student, is (Assume each way differs only by number of students watching a movie)
   (a) 68 (b) 72 (c) 84 (d) 104
5. 10 identical ball are distributed in 5 different boxes kept in a row and labeled A, B, C, D and E. Find the number of ways in which the ball can be distributed in the boxes if no two adjacent boxes remain empty.
   (a) 789 (b) 875 (c) 771 (d) 692
6. 5 different objects are to be distributed among 3 persons such that no two persons get the same number of objects. Number of ways this can be done, is
   (a) 60 (b) 90 (c) 120 (d) 150
7. The number of negative integral solution of equation $x + y + z = -12$ is
   (a) 44 (b) 55 (c) 66 (d) none of these
8. The number of ways can five people be divided into three groups is
   (a) 20 (b) 25 (c) 30 (d) 36
9. The number of ways of partitioning the set $\{a, b, c, d\}$ into one or more non empty subsets is
   (a) 14 (b) 15 (c) 16 (d) 17
10. Let y be an element of the set $A = \{1, 2, 3, 5, 6, 10, 15, 30\}$ and x_1, x_2, x_3 be integers such that $x_1x_2x_3 = y$, then the number of positive integral solutions of $x_1x_2x_3 = y$ is
    (a) 81 (b) 64 (c) 72 (d) 90

Comprehension Type

For Q. 11 to 13

There are 8 events that can be schedules in a week, then

11. The total number of ways in which the events can be scheduled is
    (a) 8^7 (b) 7^8 (c) 7! (d) 8
12. The total number of ways that the schedule has at least one event in each days of the week is
    (a) 28×5040 (b) 7! 8!
    (c) $7! \times (15!)$ (d) None of these
13. The total number of ways that these 8 event are scheduled on exactly 6 days of a week is
    (a) $210 \times 6!$ (b) $7! \times 266$
    (c) $56 \times 7!$ (d) $210 \times 7!$

For Q. 14 to 16

Let $\theta = (a_1, a_2, a_3, \ldots, a_n)$ be a given arrangement of n distinct objects $a_1, a_2, a_3, \ldots, a_n$. A derangement of θ is an arrangement of these n objects in which none of the objects occupies its original position. Let D_n be the number of derangements of the permutation θ.

14. D_n is equal to
    (a) $(n-1)D_{n-1} + D_{n-2}$ (b) $D_{n-1} + (n-1)D_{n-2}$
    (c) $n(D_{n-1} + D_{n-2})$ (d) $(n-1)(D_{n-1} + D_{n-2})$
15. The relation between D_n and D_{n-1} is given by
    (a) $D_n - nD_{n-1} = (-1)^n$ (b) $D_n - (n-1)D_{n-1} = (-1)^{n-1}$
    (c) $D_n - nD_{n-1} = (-1)^{n-1}$ (d) $D_n - D_{n-1} = (-1)^{n-1}$
16. There are 5 different colour balls and 5 boxes of colours same as those of the balls. The number of ways in which one can place the balls into the boxes, one each in a box, so that no ball goes to a box of its own colour is
    (a) 40 (b) 44 (c) 45 (d) 60

Answers Key

Single Correct Answer Type

1. (a)	2. (a)	3. (b)	4. (c)	5. (c)
6. (b)	7. (b)	8. (b)	9. (b)	10. (b)

Comprehension Type

11. (b)	12. (a)	13. (b)	14. (d)	15. (a)
16. (b)				

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CHAPTER

7

Binomial Theorem

DPP 7.1

Finding Coefficient

Single Correct Answer Type

1. Coefficient of x^6 in $((1+x)(1+x^2)^2(1+x^3)^3 \ldots (1+x^n)^n)$ is
   (a) 26 (b) 28 (c) 30 (d) 35

2. The term independent of x in the product $(4 + x + 7x^2)\left(x - \frac{3}{x}\right)^{11}$ is
   (a) $7 \cdot {}^{11}C_6$ (b) $3^6 \cdot {}^{11}C_6$
   (c) $3^5 \cdot {}^{11}C_5$ (d) $-12 \cdot 2^{11}$

3. The 13^{th} term in the expansion of $(x^2 + 2/x)^n$ is independent of x then the sum of the divisors of n is
   (a) 36 (b) 37 (c) 38 (d) 51

4. Coefficient of x^{2009} in $(1 + x + x^2 + x^3 + x^4)^{1001}(1 - x)^{1002}$ is
   (a) 0 (b) $4 \cdot {}^{1001}C_{501}$
   (c) -2009 (d) none of these

5. If the constant term in the binomial expansion of $\left(x^2 - \frac{1}{x}\right)^n$, $n \in N$ is 15 then the value of n is equal to
   (a) 6 (b) 9 (c) 12 (d) 15

6. If $p^4 + q^3 = 2(p > 0, q > 0)$, then the maximum value of term independent of x in the expansion of $\left(px^{\frac{1}{12}} + qx^{-\frac{1}{9}}\right)^{14}$ is
   (a) ${}^{14}C_4$ (b) ${}^{14}C_6$ (c) ${}^{14}C_7$ (d) ${}^{14}C_{12}$

7. In the expansion of $\left(\frac{1}{x^2} - x^3\right)^n$ $n \in N$, if the sum of the coefficients of x^5 and x^{10} is 0, then n is:
   (a) 25 (b) 20
   (c) 15 (d) None of these

8. The coefficient of t^8 in the expansion of $(1 + 2t^2 - t^3)^9$ is
   (a) 1680 (b) 2140
   (c) 2520 (d) 2730

9. The term independent of 'x' in the expansion of $\left(9x - \frac{1}{3\sqrt{x}}\right)^{18}$, $x > 0$, is α times the corresponding binomial coefficient. Then 'α' is
   (a) 3 (b) $\frac{1}{3}$ (c) $-\frac{1}{3}$ (d) 1

10. In the expansion of $\left(\frac{x}{\cos\theta} + \frac{1}{x\sin\theta}\right)^{16}$, if l_1 is the least value of the term independent of x when $\frac{\pi}{8} \le \theta \le \frac{\pi}{4}$ and l_2 is the least value of the term independent of x when $\frac{\pi}{16} \le \theta \le \frac{\pi}{8}$, then the value of $\frac{l_2}{l_1}$ is
    (a) 8 (b) 32 (c) 16 (d) 64

11. If $A_{(i,j)}$ be the coefficient of $a^i b^j c^{2010-i-j}$ in the expansion of $(a + b - c)^{2010}$, then
    (a) $A_{i,i}$ is defined for $i \ge 1010$

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(b) $A_{i,j} = A_{j,i}$

(c) $A_{2i,3i}$ is defined for $i \geq 405$

(d) $A_{0,1} = 2000$

12. The coefficient of x^{301} in the expansion of $(1+x)^{500} + x(1+x)^{499} + x^2(1+x)^{498} + \ldots + x^{500}$ is

(a) ${}^{501}C_{301}$ (b) ${}^{500}C_{301}$

(c) ${}^{501}C_{300}$ (d) none of these

13. The coefficient of x^{70} in the product $(x-1)(x^2-2)(x^3-3)(x^4-4)\ldots(x^{12}-12)$ is

(a) 4 (b) 6 (c) 8 (d) 12

14. Given $(1-x^3)^n = \sum_{k=0}^{n} a_k x^k (1-x)^{3n-2k}$ then the value of $3 \cdot a_{k-1} + a_k$ is

(a) ${}^nC_k \cdot 3^k$ (b) ${}^{(n+1)}C_k \cdot 3^k$

(c) ${}^{(n+1)}C_k \cdot 3^{k-1}$ (d) ${}^nC_{k-1} \cdot 3^k$

Answers Key

Single Correct Answer Type

1. (b) **2.** (b) **3.** (d) **4.** (a) **5.** (a)

6. (b) **7.** (c) **8.** (c) **9.** (d) **10.** (c)

11. (b) **12.** (a) **13.** (a) **14.** (b)

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DPP 7.2

Binomial Expansion and Its Applications and Determining Middle Term and Greatest Term

Single Correct Answer Type

1. The sum of the roots (real or complex) of the equation $x^{2001} + \left(\frac{1}{2} - x\right)^{2001} = 0$ is

(a) 2000 (b) 2001 (c) 1000 (d) 500

2. If the 4^{th} term of $\left\{\sqrt{x^{\frac{1}{1+\log_{10} x}}} + \sqrt[12]{x}\right\}^6$ is equal to 200, $x > 1$ and the logarithm is common logarithm, then x is not divisible by

(a) 2 (b) 5 (c) 10 (d) 4

3. The number of distinct terms in the expansion of $(x + y^2)^{13} + (x^2 + y)^{14}$ is

(a) 27 (b) 29 (c) 28 (d) 25

4. $\sum_{r=1}^{n}\sum_{p=0}^{r-1} {}^nC_r \cdot {}^rC_p \cdot 2^p$ is equal to

(a) $4^n - 3^n + 1$ (b) $4^n - 3^n - 1$
(c) $4^n - 3^n + 2$ (d) $4^n - 3^n$

5. If in the expansion of $\left(x^3 - \frac{2}{\sqrt{x}}\right)^n$ a term like x^2 exists and 'n' is a double digit number, then least value of 'n' is

(a) 10 (b) 11 (c) 12 (d) 13

6. In the binomial $(2^{1/3} + 3^{-1/3})^n$, if the ratio of the seventh term from the beginning of the expansion to the seventh term from its end is 1/6 then n =

(a) 6 (b) 9 (c) 12 (d) 15

7. The number of distinct terms in the expansion of $\left(x^3 + \frac{1}{x^3} + 1\right)^{200}$ is

(a) 201 (b) 400 (c) 401 (d) 500

8. If r^{th} and $(r+1)^{th}$ terms in the expansion of $(p+q)^n$ are equal, then $\frac{(n+1)q}{r(p+q)}$ is

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) 1 (d) 0

9. If $(3 + a\sqrt{2})^{100} + (3 + b\sqrt{2})^{100} = 7 + 5\sqrt{2}$. number of pairs (a, b) for which the equation is true is, (a, b are rational numbers)

(a) 1 (b) 6 (c) 0 (d) infinite

10. Middle term in the expansion of $(1 - 3x + 3x^2 - x^3)^{2n}$ is

(a) $\frac{(6n)!x^n}{(3n)!(3n)!}$ (b) $\frac{(6n)!x^{3n}}{(3n)!}$

(c) $\frac{(6n)}{(3n)!(3n)!}(-x)^{3n}$ (d) none of these

11. The algebraically second largest term in the expansion of $(3 - 2x)^{15}$ at $x = \frac{4}{3}$.

(a) 5 (b) 7 (c) 9 (d) 11

12. If 6^{th} term in the expansion of $\left(\frac{3}{2} + \frac{x}{3}\right)^n$ is numerically greatest, when $x = 3$, then the sum of possible integral values of 'n' is

(a) 23 (b) 24 (c) 25 (d) 26

13. Let $(5 + 2\sqrt{6})^n = p + f$ where $n \in N$ and $p \in N$ and $0 < f < 1$ then the value of, $f^2 - f + pf - p$ is

(a) a natural number (b) a negative integer
(c) a prime number (d) are irrational number

14. The sum of last 3 digits of 3^{100} is

(a) 1 (b) 2 (c) 3 (d) 4

15. The remainder when $27^{10} + 7^{51}$ is divided by 10

(a) 4 (b) 6 (c) 9 (d) 2

16. Consider the sequence $\frac{{}^nC_0}{1.2.3}, \frac{{}^nC_1}{2.3.4}, \frac{{}^nC_2}{3.4.5}, \ldots\ldots$, if $n = 50$ then greatest term is

(a) 30^{th} (b) 24^{th} (c) 26^{th} (d) 27^{th}

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Answers Key

Single Correct Answer Type

1. (d)	**2.** (d)	**3.** (c)	**4.** (d)	**5.** (a)
6. (b)	**7.** (c)	**8.** (c)	**9.** (c)	**10.** (c)
11. (b)	**12.** (c)	**13.** (b)	**14.** (a)	**15.** (d)
16. (b)				

DPP 7.3

Properties of Binomial Coefficients

Single Correct Answer Type

1. If P_n denotes the product of all the coefficients of $(1+x)^n$ and $9!P_{n+1} = 10^9 P_n$ then n is equal to
(a) 10 (b) 9 (c) 19 (d) none of these

2. If N is a prime number which divides $S = {}^{39}P_{19} + {}^{38}P_{19} + {}^{37}P_{19} + \ldots\ldots + {}^{20}P_{19}$, then the largest possible value of N among following is
(a) 41 (b) 31 (c) 37 (d) 19

3. If $\sum_{r=0}^{n}\left\{\frac{{}^{n}C_{r-1}}{{}^{n}C_r + {}^{n}C_{r-1}}\right\}^3 = \frac{25}{24}$, then n is equal to
(a) 3 (b) 4 (c) 5 (d) 6

4. If a, b, c, d be four consecutive coefficients in the binomial expansion of $(1+x)^n$, then value of the expression $\left(\left(\frac{b}{b+c}\right)^2 - \frac{ac}{(a+b)(c+d)}\right)$ (where $x > 0$ and $n \in N$) is
(a) positive (b) negative
(c) zero (d) depends on n

5. $({}^{m}C_0 + {}^{m}C_1 - {}^{m}C_2 - {}^{m}C_3) + ({}^{m}C_4 + {}^{m}C_5 - {}^{m}C_6 - {}^{m}C_7) + \ldots$ $= 0$ if and only if for some positive integer k, $m =$
(a) $4k$ (b) $4k+1$ (c) $4k-1$ (d) $4k+2$

6. The value of $\sum_{r=0}^{3n-1}(-1)^r\, {}^{6n}C_{2r+1}\, 3^r$ is
(a) 2^{3n} (b) 2^{2n-1} (c) 2^{6n-1} (d) 0

7. The coefficient of x^{50} in $(x + {}^{101}C_0)(x + {}^{101}C_1) \ldots\ldots\ldots (x + {}^{101}C_{50})$ is
(a) 4^{50} (b) 2^{50} (c) $2^{101} - 1$ (d) 2^{101}

8. In the expansion of $(1+x)^{70}$, the sum of coefficients of odd powers of x is
(a) 0 (b) 2^{69} (c) 2^{70} (d) 2^{71}

9. The sum of all the coefficients of the terms in the expansion of $(x+y+z+w)^6$ which contain x but not y, is
(a) 3^6 (b) 2^6 (c) $3^6 - 2^6$ (d) none of these

10. The value of ${}^{12}C_2 + {}^{13}C_3 + {}^{14}C_4 + \ldots + {}^{999}C_{989}$ is
(a) ${}^{1000}C_{11} - 12$ (b) ${}^{1000}C_{11} + 12$
(c) ${}^{900}C_{11} - 12$ (d) ${}^{1000}C_{989}$

11. If $(1+x+x^2)^{25} = a_0 + a_1 x + a_2 x^2 + \ldots\ldots + a_{50}\, x^{50}$, then $a_0 + a_2 + a_4 + \ldots\ldots + a_{50}$ is
(a) even
(b) odd and of the form $3n$
(c) odd and of the form $(3n-1)$
(d) odd and of the form $(3n+1)$

12. If the sum of the coefficients in the expansion of $(q+r)^{20}(1+(p-2)x)^{20}$ is equal to square of the sum of the coefficients in the expansion of $[2rqx - (r+q).y]^{10}$, where p, r, q are positive constants, then
(a) $\le p$
(b) $\frac{r+q}{2} \ge p$
(c) r, p and q are in G.P.
(d) $1/r$, $1/p$ an $1/q$ are in H.P.

13. The sum $S_n = \sum_{k=0}^{n}(-1)^k \cdot {}^{3n}C_k$, where $n = 1, 2, \ldots$ is
(a) $(-1)^n \cdot {}^{3n-1}C_{n-1}$ (b) $(-1)^n \cdot {}^{3n-1}C_n$
(c) $(-1)n \cdot {}^{3n-1}C_{n+1}$ (d) none of these

14. If $1 + (1+x) + (1+x)^2 + \ldots + (1+x)^n = \sum_{k=0}^{n} a_k x^k$, then which of the following is not true?
(a) $a_{n-2} = \frac{n(n+1)}{2}$
(b) $a_9^2 - a_8^2 = {}^{n+2}C_{10}({}^{n+1}C_{10} - {}^{n+1}C_9)$
(c) $a_p > a_{p-1}$ for $p < \frac{n}{2}$
(d) $\sum_{k=0}^{n} a_k = 2^{n+1}$

Answers Key

Single Correct Answer Type

1. (b) **2.** (a) **3.** (c) **4.** (a) **5.** (c)
6. (d) **7.** (a) **8.** (b) **9.** (c) **10.** (a)
11. (a) **12.** (b) **13.** (b) **14.** (d)

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DPP 7.4

Problems Based on Series

Single Correct Answer Type

1. Given ${}^8C_1x(1-x)^7 + 2\cdot 8C_2x^2(1-x)^6 + 3\cdot {}^8C_3x^3(1-x)^5 + \ldots + 8\cdot x^8 = ax + b$, then $a + b$ is

(a) 4 (b) 6 (c) 8 (d) 10

2. The value of $99^{50} - \frac{99}{1}\cdot 98^{50} + \frac{99.98}{1.2}97^{50} - \ldots\ldots\ldots - \frac{99.98}{1.2}\cdot 2^{50} + \frac{99}{1}\cdot 1^{50}$ is

(a) 0 (b) –1 (c) –2 (d) –3

3. Let $f(n) = \sum_{k=1}^{n} k^2({}^nC_k)^2$, then the value of $f(5)$ is equal to

(a) 1000 (b) 1250
(c) 1750 (d) 2500

4. The value of $\sum_{r=1}^{n}(-1)^{r-1}\left(\frac{r}{r+1}\right)\cdot {}^nC_r$ is

(a) $\frac{1}{n+1}$ (b) $\frac{1}{n}$ (c) $\frac{1}{n-1}$ (d) 0

5. The value of $\binom{100}{0}\binom{200}{150}+\binom{100}{1}\binom{200}{151}+\ldots\ldots+\binom{100}{50}\binom{200}{200}$ equals $\left(\text{where } \binom{n}{r} = {}^nC_r\right)$

(a) $\binom{300}{50}$ (b) $\binom{100}{50}\binom{200}{150}$

(c) $\binom{100}{50}^2$ (d) $\binom{300}{50}^2$

6. Let $t_{100} = \sum_{r=0}^{100}\frac{1}{({}^{100}C_r)^5}$ and $S_{100} = \sum_{r=0}^{100}\frac{r}{({}^{100}C_r)^5}$, then the value of $\frac{100t_{100}}{S_{100}}$ is

(a) 1 (b) 2 (c) 3 (d) 4

7. Let $S_1 = \sum\sum_{0\le i<j\le 100} C_iC_j$, $S_2 = \sum\sum_{0\le j<i\le 100} C_iC_j$ and $S_3 = \sum\sum_{0\le i=j\le 100} C_iC_j$

where C_r represents coefficient of x^r in the binomial expansion of $(1+x)^{100}$.

If $S_1 + S_2 + S_3 = a^b$ where $a, b \in N$, then the least value of $(a+b)$ is

(a) 66 (b) 72 (c) 46 (d) 52

8. ${}^{74}C_{37} - 2$ is divisible by

(a) 37^2 (b) 38 (c) 36 (d) none of these

9. If ${}^nC_0 - {}^nC_1 + {}^nC_2 - {}^nC_3 + \ldots + (-1)^r\cdot {}^nC_r = 28$, then n is equal to .....

(a) 7 (b) 8 (c) 9 (d) 11

10. If the value of ${}^nC_0 + 2\cdot {}^nC_1 + 3\cdot {}^nC_2 + \ldots\ldots + (n+1)\cdot {}^nCn = 576$, then n is

(a) 7 (b) 5 (c) 6 (d) 9

11. The value of $\binom{50}{6}-\binom{5}{1}\binom{40}{6}+\binom{5}{2}\binom{30}{6}-\binom{5}{3}\binom{20}{6}+\binom{5}{4}\binom{10}{6}$ where $\binom{n}{r}$ denotes ${}^nC_r$, is

(a) 15625 (b) 0
(c) 1000000 (d) 2250000

12. The value of the expression $\sum\sum_{0\le i<j\le n}(-1)^{i+j-1}\,{}^nC_i\cdot {}^nC_j =$

(a) ${}^{2n-1}C_n$ (b) ${}^{2n}C_n$
(c) ${}^{2n+1}C_n$ (d) none of these

13. $\sum_{m=1}^{n}\left(\sum_{k=1}^{m}\left(\sum_{p=k}^{m}{}^nC_m\cdot {}^mC_p\cdot {}^pC_k\right)\right) =$

(a) $3^n - 2^n$ (b) $4^n - 3^n$
(c) $3^n + 2^n$ (d) $4^n - 1$

Multiple Correct Answers Type

14. ${}^nC_0\,{}^{2n}C_m - {}^nC_1\,{}^{2n-2}C_m + {}^nC_2\,{}^{2n-4}C_m - \ldots\ldots =$

(a) $\binom{n}{m-n}2^{2n-m}$ if $m \ge n$

(b) 0 if $m < n$

(c) $\binom{n}{m-n}2^{2n+m}$ if $m \ge n$

(d) 1 if $m < n$

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15. Which of the following is/are correct?

(a) ${}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - \ldots\ldots\ldots - {}^{20}C_{15} = -{}^{19}C_{15}$

(b) ${}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - \ldots\ldots\ldots - {}^{20}C_{15} = -{}^{20}C_{14}$

(c) $16{}^{20}C_0 - 15\,{}^{20}C_1 + 14{}^{20}C_2 - \ldots\ldots\ldots + 2{}^{20}C_{14} - {}^{20}C_{15} = {}^{19}C_{14}$

(d) $16{}^{20}C_0 - 15\,{}^{20}C_1 + 14{}^{20}C_2 - \ldots\ldots\ldots + 2{}^{20}C_{14} - {}^{20}C_{15} = {}^{18}C_{15}$

16. The value of $\sum_{k=0}^{7}\left[\frac{\binom{7}{k}}{\binom{14}{k}}\sum_{r=k}^{14}\binom{r}{k}\binom{14}{r}\right]$, where $\binom{n}{r}$ denotes ${}^{n}C_{r}$, is

(a) 6^7 (b) greater than 7^6

(c) 8^7 (d) greater than 7^8

17. The value of ${}^{1000}C_{50} + {}^{999}C_{49} + {}^{998}C_{48} + \ldots\ldots + {}^{950}C_{0}$ is

(a) ${}^{1001}C_{50}$ (b) ${}^{1002}C_{951} - {}^{1001}C_{51}$

(c) ${}^{1001}C_{951}$ (d) ${}^{1002}C_{51} - {}^{1001}C_{95}$

Answers Key

Single Correct Answer Type

1. (c)	**2.** (a)	**3.** (c)	**4.** (a)	**5.** (a)
6. (b)	**7.** (a)	**8.** (a)	**9.** (c)	**10.** (a)
11. (d)	**12.** (a)	**13.** (b)		

Multiple Correct Answers Type

14. (a, b) **15.** (a, d) **16.** (a, b)

17. (a, b, c, d)

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DPP 7.5

Miscellaneous, Binomial Expansion for Any Index

Single Correct Answer Type

1. If $\frac{(1-3x)^{1/2}+(1-x)^{5/3}}{\sqrt{4-x}}$ is approximately equal to $a+bx$ for small values of x, then $(a, b) =$

(a) $\left(1, \frac{35}{24}\right)$ (b) $\left(1, -\frac{35}{24}\right)$

(c) $\left(2, \frac{35}{12}\right)$ (d) $\left(2, -\frac{35}{12}\right)$

2. The value of $1 + \frac{1}{3^2} + \frac{1.4}{1.2}\frac{1}{3^4} + \frac{1.4.7}{1.2.3}\frac{1}{3^6} + \ldots$ is

(a) $\left(\frac{3}{2}\right)^{\frac{1}{3}}$ (b) $\left(\frac{5}{4}\right)^{\frac{1}{3}}$

(c) $\left(\frac{3}{2}\right)^{\frac{1}{6}}$ (d) None of these

3. Coefficient of $x^{2^{m+1}}$ in the expansion of $\frac{1}{(1+x)(1+x^2)(1+x^4)(1+x^8)\ldots(1+x^{2^m})}$ $(|x|<1)$ is

(a) 0 (b) 1 (c) 2^m (d) none of these

4. Let $(2x^2+3x+4)^{10} = \sum_{r=0}^{20} a_r x^r$, then the value of $\frac{a_7}{a_{13}}$, is

(a) 6 (b) 8 (c) 12 (d) 16

5. ${}^{30}C_0 \cdot {}^{20}C_{10} + {}^{31}C_1 \cdot {}^{19}C_{10} + {}^{32}C_2 \cdot 18C_{10} + \ldots\ldots {}^{40}C_{10} \cdot {}^{10}C_{10}$ is equal to

(a) ${}^{51}C_{41}$ (b) ${}^{50}C_{40}$

(c) ${}^{51}C_{21}$ (d) ${}^{50}C_{40}$

Comprehension Type

For Q. 6 and 7

Consider a G.P. with first term $(1+x)^n$, $|x| < 1$, common ratio $\frac{1+x}{2}$ and number of terms $(n+1)$. Let 'S' be sum of all the terms of the G.P., then

6. The coefficient of x^n in 'S' is

(a) 2^n (b) 2^{n+1} (c) 2^{2n} (d) 2^{2n+1}

7. $\sum_{r=0}^{n} {}^{n+r}C_r \left(\frac{1}{2}\right)^r$ equals

(a) $(3/4)^n$ (b) 1 (c) 2^n (d) 3^n

For Q. 8 to 10

A path of length n is a sequence of points (x_1, y_1), (x_2, y_2), …, (x_n, y_n) with integer coordinates such that for all i between 1 and $n-1$ both inclusive,

either $x_{i+1} = x_i + 1$ and $y_{i+1} = y_i$ (in which case we say the i^{th} step is rightward)

or $x_{i+1} = x_i$ and $y_{i+1} = y_i + 1$ (in which case we say that the i^{th} step is upward).

This path is said to start at (x_1, y_1) and end at (x_n, y_n). Let $P(a, b)$, for a and b non-negative integers, denotes the number of paths that start at (0, 0) and end at (a, b).

8. The value of $\sum_{i=0}^{10} P(i, 10-i)$ is

(a) 1024 (b) 512 (c) 256 (d) 128

9. Number of ordered pairs (i, j) where $i \neq j$ for which $P(i, 100-i) = P(j, 100-j)$ is

(a) 50 (b) 99 (c) 100 (d) 101

10. The sum $P(43, 4) + \sum_{j=1}^{5} P(49-j, 3)$ is equal to

(a) $P(4, 48)$ (b) $P(3, 49)$

(c) $P(4, 47)$ (d) $P(5, 47)$

For Q. 11 to 13

The expressions $1+x$, $1+x+x^2$, $1+x+x^2+x^3$,… $1+x+x^2+\ldots+x^{20}$ are multiplied together and the terms of the product thus obtained are arranged in increasing powers of x in the form of $a_0 + a_1x + a_2x^2 + \ldots$, then,

11. Number of terms in the product

(a) 200 (b) 211

(c) 231 (d) none of these

12. Sum of coefficients of even powers of x is

(a) 20! (b) 21!

(c) $\frac{21!}{2}$ (d) 19!

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13. The value of $\frac{a_r}{a_{n-r}}$, where n is the degree of the product.

(a) 2 (b) 1
(c) 1/2 (d) depends on r

For Q. 14 to 16

If $(1 + px + x^2)^n = 1 + a_1x + a_2x^2 + \ldots + a_{2n}x^{2n}$.

14. Which of the following is true for $1 < r < 2n$

(a) $(np + pr)a_r = (r + 1)a_{r+1} + (r - 1)a_{r-1}$
(b) $(np - pr)a_r = (r + 1)a_{r+1} + (r - 1 - 2n)a_{r-1}$
(c) $(np - pr)a_r = (r + 1)a_{r+1} + (r - 1 - n)a_{r-1}$
(d) $(2np + pr)a_r = (r + 1 + n)a_{r+1} + (r + 1 - n)a_{r-1}$

15. The remainder obtained when $a_1 + 5a_2 + 9a_3 + 13a_4 + \ldots + (8n - 3)a_{2n}$ is divided by $(p + 2)$ is

(a) 1 (b) 2 (c) 3 (d) 0

16. The value of $a_1 + 3a_2 + 5a_3 + 7a_4 + \ldots + (4n - 1)a_{2n}$ when $p = -3$ and $n \in$ even is

(a) n (b) $2n - 1$ (c) $2n - 2$ (d) $2n$

Answers Key

Single Correct Answer Type

1. (b) **2.** (a) **3.** (b) **4.** (b) **5.** (a)

Comprehension Type

6. (a) **7.** (c) **8.** (a) **9.** (c) **10.** (a)
11. (b) **12.** (c) **13.** (b) **14.** (b) **15.** (c)
16. (d)

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CHAPTER

8

Determinant

DPP 8.1

Expansion of Determinant

Single Correct Answer Type

1. $\begin{vmatrix} 1 & \cos\alpha & \cos\beta \\ \cos\alpha & 1 & \cos\gamma \\ \cos\beta & \cos\gamma & 1 \end{vmatrix} = \begin{vmatrix} 0 & \cos\alpha & \cos\beta \\ \cos\alpha & 0 & \cos\beta \\ \cos\beta & \cos\gamma & 0 \end{vmatrix}$

if $\cos^2\alpha + \cos^2\beta + \cos^2\gamma =$

(a) 1 (b) 2 (c) 3/2 (d) 1/2

2. If α, β, γ are roots of the equation $x^2(px + q) = r(x + 1)$, then the value of determinant $\begin{vmatrix} 1+\alpha & 1 & 1 \\ 1 & 1+\beta & 1 \\ 1 & 1 & 1+\gamma \end{vmatrix}$ is

(a) $\alpha\beta\gamma$ (b) $1+\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}$

(c) 0 (d) none of these

3. If $\omega \neq 1$ is cube root of unity and $x + y + z \neq 0$, then

$$\begin{vmatrix} \frac{x}{1+\omega} & \frac{y}{\omega+\omega^2} & \frac{z}{\omega^2+1} \\ \frac{y}{\omega+\omega^2} & \frac{z}{\omega^2+1} & \frac{x}{1+\omega} \\ \frac{z}{\omega^2+1} & \frac{x}{1+\omega} & \frac{y}{\omega+\omega^2} \end{vmatrix} = 0, \text{ if}$$

(a) $x^2 + y^2 + z^2 = 0$

(b) $x + y\omega + z\omega^2 = 0$ or $x = y = z$

(c) $x \neq y \neq z \neq 0$

(d) $x = 2y = 3z$

4. If $a = \cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}$, then $\begin{vmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{vmatrix}$ is

(a) purely real (b) purely imaginary

(c) 0 (d) None of these

5. If 'α' is a root of $x^4 = 1$ with negative principal argument, then the principal argument of $\Delta(\alpha) =$

$$\begin{vmatrix} 1 & 1 & 1 \\ \alpha^n & \alpha^{n+1} & \alpha^{n+3} \\ \frac{1}{\alpha^{n+1}} & \frac{1}{\alpha^n} & 0 \end{vmatrix} \text{ is}$$

(a) $\frac{5\pi}{14}$ (b) $-\frac{3\pi}{4}$ (c) $\frac{\pi}{4}$ (d) $-\frac{\pi}{4}$

6. $\Delta_1 = \begin{vmatrix} x & b & b \\ a & x & b \\ a & a & x \end{vmatrix}$ and $\Delta_2 = \begin{vmatrix} x & b \\ a & x \end{vmatrix}$ are the given determinants, then

(a) $\Delta_1 = 3(\Delta_2)^2$ (b) $\frac{d}{dx}(\Delta_1) = 3\Delta_2$

(c) $\frac{d}{dx}(\Delta_1) = 3(\Delta_2)^2$ (d) $\Delta_1 = 3\Delta_2^{3/2}$

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7. If $a^2+b^2+c^2+ab+bc+ca \le 0\ \forall\ a, b, c \in R$, then value of the determinant $\begin{vmatrix} (a+b+2)^2 & a^2+b^2 & 1 \\ 1 & (b+c+2)^2 & b^2+c^2 \\ c^2+a^2 & 1 & (c+a+2)^2 \end{vmatrix}$ equals

(a) 65 (b) $a^2+b^2+c^2+31$

(c) $4(a^2+b^2+c^2)$ (d) 0

8. Product of roots of equation $\begin{vmatrix} 1+2x & 1 & 1-x \\ 2-x & 2+x & 3+x \\ x & 1+x & 1-x^2 \end{vmatrix} = 0$ is

(a) 1/2 (b) 3/4 (c) 4/3 (d) 1/4

Comprehension Type

For Q. 9 to 10

A 3×3 determinant has entries either 1 or -1.

Let S_3 = set of all determinants which contain determinants such that product of elements of any row or any column is -1.

For example $\begin{vmatrix} 1 & -1 & 1 \\ 1 & 1 & -1 \\ -1 & 1 & 1 \end{vmatrix}$ is an element of the set S_3.

9. Number of elements of the set S_3 =

(a) 10 (b) 16 (c) 12 (d) 18

10. Number of elements of the set S_n =

(a) 2^n (b) 2^{n-1} (c) 2^{2n} (d) $2^{(n-1)^2}$

Answers Key

Single Correct Answer Type

1. (a) **2.** (c) **3.** (b) **4.** (b) **5.** (b)
6. (b) **7.** (a) **8.** (a)

Comprehension Type

9. (b) **10.** (d)

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DPP 8.2

Properties of Determinant (Level 1)

Single Correct Answer Type

1. If x, y, z are non-zero real numbers and $\begin{vmatrix} 1+x & 1 & 1 \\ 1+y & 1+2y & 1 \\ 1+z & 1+z & 1+3z \end{vmatrix} = 0$, then $-\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)$ is equal to

(a) 0 (b) 1 (c) 3 (d) 6

2. If $Y = SX$, $Z = tX$ all the variables being differentiable functions of x and lower suffices denote the derivative with respect to x and $\begin{vmatrix} X & Y & Z \\ X_1 & Y_1 & Z_1 \\ X_2 & Y_2 & Z_2 \end{vmatrix} \div \begin{vmatrix} S_1 & t_1 \\ S_2 & t_2 \end{vmatrix} = X^n$, then $n =$

(a) 1 (b) 2 (c) 3 (d) 4

3. If $w \neq 1$ is a cube root of unity and $\Delta = \begin{vmatrix} x+w^2 & w & 1 \\ w & w^2 & 1+x \\ 1 & x+w & w^2 \end{vmatrix}$ $= 0$, then value of x is

(a) 0 (b) 1 (c) -1 (d) none of these

4. Let $|A| = |a_{ij}|_{3\times 3} \neq 0$. Each element a_{ij} is multiplied by k^{i-j}. Let $|B|$ the resulting determinant, where $k_1|A| + k_2|B| = 0$. Then $k_1 + k_2 =$

(a) 1 (b) -1 (c) 0 (d) 2

5. If α, β, γ are the roots of $x^3 + px^2 + q = 0$, where $q \neq 0$, then $\Delta = \begin{vmatrix} 1/\alpha & 1/\beta & 1/\gamma \\ 1/\beta & 1/\gamma & 1/\alpha \\ 1/\gamma & 1/\alpha & 1/\beta \end{vmatrix}$ equals

(a) $-p/q$ (b) $1/q$
(c) p^2/q (d) None of these

6. If $a - 2b + c = 1$, then the value of $\begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix}$ is

(a) x (b) $-x$ (c) -1 (d) 1

7. Let $x > 0, y > 0, z > 0$ are respectively the 2nd, 3rd, 4th terms of a G.P and $\Delta = \begin{vmatrix} x^k & x^{k+1} & x^{k+2} \\ y^k & y^{k+1} & y^{k+2} \\ z^k & z^{k+1} & z^{k+2} \end{vmatrix} = (r-1)^2\left(1-\frac{1}{r^2}\right)$

(where r is the common ratio), then

(a) $k = -1$ (b) $k = 1$
(c) $k = 0$ (d) None of these

8. If $a, b, c, d > 0$; $x \in R$ and $(a^2 + b^2 + c^2)x^2 - 2(ab + bc + cd)x + b^2 + c^2 + d^2 \leq 0$, then $\begin{vmatrix} 33 & 14 & \log a \\ 65 & 27 & \log b \\ 97 & 40 & \log c \end{vmatrix} =$

(a) 1 (b) -1
(c) 0 (d) none of these

9. $\begin{vmatrix} {}^xC_r & {}^xC_{r+1} & {}^xC_{r+2} \\ {}^yC_r & {}^yC_{r+1} & {}^yC_{r+2} \\ {}^zC_r & {}^zC_{r+1} & {}^zC_{r+2} \end{vmatrix} - \begin{vmatrix} {}^xC_r & {}^{x+1}C_{r+1} & {}^{x+2}C_{r+2} \\ {}^yC_r & {}^{y+1}C_{r+1} & {}^{y+2}C_{r+2} \\ {}^zC_r & {}^{z+1}C_{r+1} & {}^{z+2}C_{r+2} \end{vmatrix}$

(a) 0 (b) 2^n
(c) ${}^{x+y+z}C_r$ (d) ${}^{x+y+z}C_{r+2}$

10. If $\begin{vmatrix} {}^9C_4 & {}^9C_5 & {}^{10}C_r \\ {}^{10}C_6 & {}^{10}C_7 & {}^{11}C_{r+2} \\ {}^{11}C_8 & {}^{11}C_9 & {}^{12}C_{r+4} \end{vmatrix} = 0$, then the value of r is equal to

(a) 3 (b) 4 (c) 5 (d) 6

11. If either of the two P, Q and R are equal and $P + Q + R = 180°$, then the value of $\begin{vmatrix} 1 & 1+\sin P & \sin P(1+\sin P) \\ 1 & 1+\sin Q & \sin Q(1+\sin Q) \\ 1 & 1+\sin R & \sin R(1+\sin R) \end{vmatrix}$ is

(a) 0 (b) 1
(c) $\sin(P+Q+R)$ (d) $\sin P \sin Q \sin R$

12. In a triangle ABC, if a, b, c are the sides opposite to angles A, B, C respectively, then the value of $\begin{vmatrix} b\cos C & a & c\cos B \\ c\cos A & b & a\cos C \\ a\cos B & c & b\cos A \end{vmatrix}$ is

(a) 1 (b) -1
(c) 0 (d) $a\cos A + b\cos B + c\cos C$

13. If $a = 1 + 2 + 4 + \ldots$ up to n terms
$b = 1 + 3 + 9 + \ldots$ up to n terms
and $c = 1 + 5 + 25 + \ldots$ up to n terms.
then $\Delta = \begin{vmatrix} a & 2b & 4c \\ 2 & 2 & 2 \\ 2^n & 3^n & 5^n \end{vmatrix} =$

(a) $(30)^n$ (b) $(10)^n$ (c) 0 (d) $2^n + 3^n + 5^n$

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14. If $a_1, a_2, a_3, 5, 4, a_6, a_7, a_8, a_9$ are in H.P., and the value of the determinant $\begin{vmatrix} a_1 & a_2 & a_3 \\ 5 & 4 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix}$ is D, then the value of $21D/10$ is

(a) 4 (b) 5
(c) 6 (d) 7

15. The value of

$$\begin{vmatrix} 1/c & 1/c & -(a+b)/c^2 \\ -(b+c)/a^2 & 1/a & 1/a \\ -b(b+c)/a^2c & (a+2b+c)/ac & -b(a+b)/ac^2 \end{vmatrix}$$ is

(a) dependent on a, b, c (b) dependent on a
(c) dependent on b (d) independent of a, b and c

Answers Key

Single Correct Answer Type

1. (c) **2.** (c) **3.** (a) **4.** (c) **5.** (d)
6. (c) **7.** (a) **8.** (c) **9.** (a) **10.** (c)
11. (a) **12.** (c) **13.** (c) **14.** (b) **15.** (a)

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DPP 8.3

Properties of Determinant (Level 2)

Single Correct Answer Type

1. The equation $\begin{vmatrix}(1+x)^2 & (1-x)^2 & -(2+x^2)\\ 2x+1 & 3x & 1-5x\\ x+1 & 2x & 2-3x\end{vmatrix} + \begin{vmatrix}(1+x)^2 & 2x+1 & x+1\\ (1-x)^2 & 3x & 2x\\ 1-2x & 3x-2 & 2x-3\end{vmatrix} = 0$

(a) has no real solution
(b) has 4 real solutions
(c) has two real and two non-real solutions
(d) has infinite number of solutions , real or non-real

2. Let $\Delta_1 = \begin{vmatrix}ap^2 & 2ap & 1\\ aq^2 & 2aq & 1\\ ar^2 & 2ar & 1\end{vmatrix}$ and $\Delta_2 = \begin{vmatrix}apq & a(p+q) & 1\\ aqr & a(q+r) & 1\\ arp & a(r+p) & 1\end{vmatrix}$, then

(a) $\Delta_1 = \Delta_2$ (b) $\Delta_2 = 2\Delta_1$
(c) $\Delta_1 = 2\Delta_2$ (d) $\Delta_1 + 2\Delta_2 = 0$

3. Area of triangle whose vertices are $(a, a^2)(b, b^2)(c, c^2)$ is $\frac{1}{2}$, and area of another triangle whose vertices are (p, p^2), (q, q^2) and (r, r^2) is 4, then the value of $\begin{vmatrix}(1+ap)^2 & (1+bp)^2 & (1+cp)^2\\ (1+aq)^2 & (1+bq)^2 & (1+cq)^2\\ (1+ar)^2 & (1+br)^2 & (1+cr)^2\end{vmatrix}$ is

(a) 2 (b) 4 (c) 8 (d) 16

4. The value of $\begin{vmatrix}\beta\gamma & \beta\gamma'+\beta'\gamma & \beta'\gamma'\\ \gamma\alpha & \gamma\alpha'+\gamma'\alpha & \gamma'\alpha'\\ \alpha\beta & \alpha\beta'+\alpha'\beta & \alpha'\beta'\end{vmatrix}$ is

(a) $(\alpha\beta' - \alpha'\beta)(\beta\gamma' - \beta'\gamma)(\gamma\alpha' - \gamma'\alpha)$
(b) $(\alpha\alpha' - \beta\beta')(\beta\beta' - \gamma\gamma')(\gamma\gamma' - \alpha\alpha')$
(c) $(\alpha\beta' + \alpha'\beta)(\beta\gamma' + \beta'\gamma)(\gamma\alpha' + \gamma'\alpha)$
(d) none of these

5. If $\begin{vmatrix}a & b & 1\\ b & c & 1\\ c & a & 1\end{vmatrix} = 2010$ and if $\begin{vmatrix}c-a & c-b & ab\\ a-b & a-c & bc\\ b-c & b-a & ca\end{vmatrix} \begin{vmatrix}c-a & c-b & c^2\\ a-b & a-c & a^2\\ b-c & b-a & b^2\end{vmatrix} = p$, then the number of positive divisors of p is

(a) 36 (b) 49 (c) 64 (d) 81

6. If $\begin{vmatrix}a & l & m\\ l & b & n\\ m & n & c\end{vmatrix}\begin{vmatrix}bc-n^2 & mn-lc & ln-bm\\ mn-lc & ac-m^2 & lm-an\\ ln-bm & lm-an & ab-l^2\end{vmatrix} = 64$, then the value of $\begin{vmatrix}2a+3l & 3l+5m & 5m+4a\\ 2l+3b & 3b+5n & 5n+4l\\ 2m+3n & 3n+5c & 5c+4m\end{vmatrix}$ equals

(a) 120 (b) 240 (c) 360 (d) 480

7. The value of $\begin{vmatrix}x^2+y^2 & ax+by & x+y\\ ax+by & a^2+b^2 & a+b\\ x+y & a+b & 2\end{vmatrix}$ depends on

(a) a (b) b (c) x (d) none of these

8. If $u = ax + by + cz$, $v = ay + bz + cx$, $w = ax + bx + cy$, then the value of $\begin{vmatrix}a & b & c\\ b & c & a\\ c & a & b\end{vmatrix} \times \begin{vmatrix}x & y & z\\ y & z & x\\ z & x & y\end{vmatrix}$ is

(a) $u^2 + v^2 + w^2 - 2uvw$ (b) $u^3 + v^3 + w^3 - 3uvw$
(c) 0 (d) none of these

9. If the number of positive integral solutions of $u + v + w = n$ be denoted by P_n, then the absolute value of $\begin{vmatrix}P_n & P_{n+1} & P_{n+2}\\ P_{n+1} & P_{n+2} & P_{n+3}\\ P_{n+2} & P_{n+3} & P_{n+4}\end{vmatrix}$ is

(a) -1 (b) 2 (c) 3 (d) 4

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10. If $f(x)$, $h(x)$ are polynomials of degree 4 and $\begin{vmatrix} f(x) & g(x) & h(x) \\ a & b & c \\ p & q & r \end{vmatrix} = mx^4+nx^3+rx^2+sx+t$ be an identity in x, then $\begin{vmatrix} f'''(0)-f''(0) & g'''(0)-g''(0) & h'''(0)-h''(0) \\ a & b & c \\ p & q & r \end{vmatrix}$ is

(a) $2(3n-r)$ (b) $2(2n-3r)$
(c) $3(n-2r)$ (d) none of these

11. If $\Delta(x)=\begin{vmatrix} x-2 & (x-1)^2 & x^3 \\ x-1 & x^2 & (x+1)^3 \\ x & (x+1)^2 & (x+2)^3 \end{vmatrix}$, then coefficient of x in $\Delta(x)$ is

(a) –4 (b) –2 (c) –6 (d) 0

12. If $f(x)=\begin{vmatrix} \sin x & \cos x & \tan x \\ x^3 & x^2 & x \\ 2x & 1 & x \end{vmatrix}$, then $\lim_{x\to 0}\frac{f(x)}{x^2}=$

(a) 0 (b) 3 (c) 2 (d) 1

Multiple Correct Answers Type

13. If $x\in R, a_i, b_i, c_i \in R$ for $i=1,2,3$ and $\begin{vmatrix} a_1+b_1x & a_1x+b_1 & c_1 \\ a_2+b_2x & a_2x+b_2 & c_2 \\ a_3+b_3x & a_3x+b_3 & c_3 \end{vmatrix} = 0$, then which of the following may be true?

(a) $x=1$ (b) $x=-1$
(c) $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}=0$ (d) none of these

14. If a_i, i = 1, 2, ..., 9 are perfect odd squares, then $\begin{vmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix}$ is always a multiple of

(a) 4 (b) 7 (c) 16 (d) 64

15. The value of the determinant $\begin{vmatrix} \cos(\theta+\alpha) & -\sin(\theta+\alpha) & \cos 2\alpha \\ \sin\theta & \cos\theta & \sin\alpha \\ -\cos\theta & \sin\theta & \lambda\cos\alpha \end{vmatrix}$ is

(a) independent of θ for all $\lambda \in R$
(b) independent of θ and α when $\lambda = 1$
(c) independent of θ and α when $\lambda = -1$
(d) independent of λ for all θ

Answers Key

Single Correct Answer Type

1. (d) **2.** (d) **3.** (d) **4.** (a) **5.** (d)
6. (c) **7.** (d) **8.** (b) **9.** (a) **10.** (a)
11. (b) **12.** (d)

Multiple Correct Answers Type

13. (a, b, c) **14.** (a, c, d) **15.** (a, c)

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DPP 8.4

System of Equations

Single Correct Answer Type

1. If the system of linear equations
$x + 2ay + az = 0$
$x + 3by + bz = 0$
$x + 4cy + cz = 0$
has a non zero solutions, then a, b, c are in
(a) A.P. (b) G.P.
(c) H.P. (d) satisfies $a + 2b + 3c = 0$

2. The equations $(\lambda - 1)x + (3\lambda + 1)y + 2\lambda z = 0$, $(\lambda - 1)x + (4\lambda - 2)y + (\lambda + 3)z = 0$ and $2x + (3\lambda + 1)y + 3(\lambda - 1)z = 0$
gives non-trivial solution for some values of λ, then the ratio $x : y : z$ when λ has the smallest of these values:
(a) 3 : 2 : 1 (b) 3 : 3 : 2 (c) 1 : 3 : 1 (d) 1 : 1 : 1

3. The system of homogeneous equations
$tx + (t+1)y + (t-1)z = 0$, $(t+1)x + ty + (t+2)z = 0$, $(t-1)x + (t+2)y + tz = 0$ has a non-trivial solution for
(a) exactly three real values of t
(b) exactly two real value of t
(c) exactly one real value of t
(d) infinite number of values of t

4. If a, b, c are non-zeros, then the system of equations
$(\alpha + a)x + \alpha y + \alpha z = 0$
$\alpha x + (\alpha + b)y + \alpha z = 0$
$\alpha x + \alpha y + (\alpha + c)z = 0$
has a non-trivial solution if
(a) $2\alpha = a + b + c$ (b) $\alpha^{-1} = a + b + c$
(c) $\alpha + a + b + c = 1$ (d) $\alpha^{-1} = -(a^{-1} + b^{-1} + c^{-1})$

5. The values of θ, λ for which the following equations $\sin\theta x - \cos\theta y + (\lambda + 1)z = 0$; $\cos\theta x + \sin\theta y - \lambda z = 0$; $\lambda x + (\lambda + 1)y + \cos\theta z = 0$
have non trivial solution, is
(a) $\theta = n\pi, \lambda \in R - \{0\}$
(b) $\theta = 2n\pi$, λ is any rational number
(c) $\theta = (2n + 1)\pi, \lambda \in R^+, n \in I$
(d) $\theta = (2n + 1)\frac{\pi}{2}, \lambda \in R, n \in I$

6. If the system of equations
$x - 2y + z = a$
$2x + y - 2z = b$
and $x + 3y - 3z = c$
have at least one solution, then
(a) $a + b + c = 0$ (b) $a - b + c = 0$
(c) $-a + b + c = 0$ (d) $a + b - c = 0$

7. If A, B, C are the angles of a triangle, the system of equations
$(\sin A)x + y + z = \cos A$, $x + (\sin B)y + z = \cos B$, $x + y + (\sin C)z = 1 - \cos C$ has
(a) No solution
(b) Unique solution
(c) Infinitely many solutions
(d) Finitely many solutions

Multiple Correct Answers Type

8. A solution set of the equations $x + 2y + z = 1$, $x + 3y + 4z = k$, $x + 5y + 10z = k^2$ is
(a) $(1 + 5\lambda, -3\lambda, \lambda)$ (b) $(5\lambda - 1, 1 - 3\lambda, \lambda)$
(c) $(1 + 6\lambda, -2\lambda, \lambda)$ (d) $(1 - 6\lambda, \lambda, \lambda)$

9. Consider the system of equations: $x\sin\theta - 2y\cos\theta - az = 0$, $x + 2y + z = 0$, $-x + y + z = 0$, $\theta \in R$
(a) The given system will have infinite solutions for $a = 2$
(b) The number of integer values of a is 3 for the system to have nontrivial solutions.
(c) For $a = 1$ there exists θ for which the system will have infinite solutions
(d) For $a = 3$ there exists θ for which the system will have unique solution

Answers Key

Single Correct Answer Type

1. (c) **2.** (d) **3.** (c) **4.** (d) **5.** (d)
6. (b) **7.** (b)

Multiple Correct Answers Type

8. (a, b) **9.** (b, c, d)

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CHAPTER

9

Matrices

DPP 9.1

Matrices and Its Properties (Level 1)

Single Correct Answer Type

1. If $A = \begin{bmatrix} 0 & c & -b \\ -c & 0 & a \\ b & -a & 0 \end{bmatrix}$ and $B = \begin{bmatrix} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{bmatrix}$, then $(A+B)^2 =$
(a) A (b) B (c) I (d) $A^2 + B^2$

2. If the value of $\prod_{k=1}^{50} \begin{bmatrix} 1 & 2k-1 \\ 0 & 1 \end{bmatrix}$ is equal to $\begin{bmatrix} 1 & r \\ 0 & 1 \end{bmatrix}$ then r is equal to
(a) 62500 (b) 2500 (c) 1250 (d) 12500

3. A square matrix P satisfies $P^2 = I - P$ where I is identity matrix. If $P^n = 5I - 8P$, then n is
(a) 4 (b) 5 (c) 6 (d) 7

4. A and B are two square matrices such that $A^2B = BA$ and if $(AB)^{10} = A^kB^{10}$, then k is
(a) 1001 (b) 1023 (c) 1042 (d) none of these

5. If matrix $A = [a_{ij}]_{3\times3}$, matrix $B = [b_{ij}]_{3\times3}$, where $a_{ij} + a_{ji} = 0$ and $b_{ij} - b_{ji} = 0\ \forall\ i, j$, then $A^4 \cdot B^3$ is
(a) Singular (b) Zero matrix
(c) Symmetric (d) Skew-Symmetric matrix

6. If $A\begin{pmatrix} 1 & 3 & 4 \\ 3 & -1 & 5 \\ -2 & 4 & -3 \end{pmatrix} = \begin{pmatrix} 3 & -1 & 5 \\ 1 & 3 & 4 \\ +4 & -8 & 6 \end{pmatrix}$, then $A =$

(a) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$ (b) $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

(c) $\begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -2 \end{pmatrix}$ (d) $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -2 \end{pmatrix}$

7. Let $A = \begin{bmatrix} -5 & -8 & -7 \\ 3 & 5 & 4 \\ 2 & 3 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$. If AB is a scalar multiple of B, then the value of $x + y$ is
(a) -1 (b) -2 (c) 1 (d) 2

8. $A = \begin{bmatrix} a & b \\ b & -a \end{bmatrix}$ and $MA = A^{2m}$, $m \in N$ for some matrix M, then which one of the following is correct?
(a) $M = \begin{bmatrix} a^{2m} & b^{2m} \\ b^{2m} & -a^{2m} \end{bmatrix}$

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(b) $M = (a^2 + b^2)^m \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(c) $M = (a^m + b^m) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(d) $M = (a^2 + b^2)^{m-1} \begin{bmatrix} a & b \\ b & -a \end{bmatrix}$

9. If $A = [a_{ij}]_{m \times n}$ and $a_{ij} = (i^2 + j^2 - ij)(j - i)$, n is odd, then which of the following is not the value of $Tr(A)$

(a) 0 (b) $|A|$ (c) $2|A|$ (d) none of these

10. $|A - B| \neq 0$, $A^4 = B^4$, $C^3A = C^3B$, $B^3A = A^3B$, then $|A^3 + B^3 + C^3| =$

(a) 0 (b) 1 (c) $3|A|^3$ (d) 6

11. If $AB + BA = O$, then which of the following is equivalent to $A^3 - B^3$

(a) $(A - B)(A^2 + AB + B^2)$ (b) $(A - B)(A^2 - AB - B^2)$

(c) $(A + B)(A^2 - AB - B^2)$ (d) $(A + B)(A^2 + AB - B^2)$

12. A, B, C are three matrices of the same order such that any two are symmetric and the 3rd one is skew symmetric. If $X = ABC + CBA$ and $Y = ABC - CBA$, then $(XY)^T$ is

(a) symmetric (b) skew symmetric

(c) $I - XY$ (d) $-YX$

13. If A and P are different matrices of order n satisfying $A^3 = P^3$ and $A^2P = P^2A$ (where $|A - P| \neq 0$) then $|A^2 + P^2|$ is equal to

(a) n (b) 0 (c) $|A||P|$ (d) $|A + P|$

Answers Key

Single Correct Answer Type

1. (d) **2.** (b) **3.** (c) **4.** (b) **5.** (a)

6. (d) **7.** (b) **8.** (d) **9.** (d) **10.** (a)

11. (c) **12.** (d) **13.** (b)

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DPP 9.2

Matrices and Its Properties (Level 2)

Single Correct Answer Type

1. Let A, B are square matrices of same order satisfying $AB = A$ and $BA = B$ then $(A^{2010} + B^{2010})^{2011}$ equals.
   (a) $A + B$ (b) $2010(A + B)$
   (c) $2011(A + B)$ (d) $2^{2011}(A + B)$
2. The number of 2×2 matrices A, that are there with the elements as real numbers satisfying $A + A^T = I$ and $AA^T = I$ is
   (a) zero (b) one (c) two (d) infinite
3. If the orthogonal square matrices A and B of same size satisfy $\det A + \det B = 0$ then the value of $\det (A + B)$
   (a) -1 (b) 1
   (c) 0 (d) none of these
4. If $A = \begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}, C = ABA^T$, then $A^T C^n A$ equals to $(n \in I^+)$
   (a) $\begin{bmatrix} -n & 1 \\ 1 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & -n \\ 0 & 1 \end{bmatrix}$
   (c) $\begin{bmatrix} 0 & 1 \\ 1 & -n \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 \\ -n & 1 \end{bmatrix}$
5. Let A be a 3×3 matrix given by $A = (a_{ij})_{3\times 3}$. If for every column vector X satisfies $X'AX = 0$ and $a_{12} = 2008$, $a_{13} = 1010$ and $a_{23} = -2012$. Then the value of $a_{21} + a_{31} + a_{32} =$
   (a) -6 (b) 2006 (c) -2006 (d) 0
6. A and B are two non-singular matrices such that $A^6 = I$ and $AB^2 = BA (B \neq I)$. A value of k so that $B^k = I$
   (a) 31 (b) 32 (c) 64 (d) 63
7. Let A be a 2×3 matrix, whereas B be a 3×2 matrix. If $\det. (AB) = 4$, then the value of $\det. (BA)$ is
   (a) -4 (b) 2 (c) -2 (d) 0
8. Let A be a square matrix of order 3 so that sum of elements of each row is 1. Then the sum elements of matrix A^2 is
   (a) 1 (b) 3 (c) 0 (d) 6
9. A and B be 3×3 matrices such that $AB + A + B = 0$, then
   (a) $(A + B)^2 = A^2 + 2AB + B^2$
   (b) $|A| = |B|$
   (c) $A^2 = B^2$
   (d) none of these
10. If $(A + B)^2 = A^2 + B^2$ and $|A| \neq 0$, then $|B| =$ (where A and B are matrices of odd order)
    (a) 2 (b) -2 (c) 1 (d) 0

Multiple Correct Answers Type

11. If $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$, then
    (a) $A^3 - A^2 = A - I$ (b) $\text{Det}(A^{2010} - I) = 0$
    (c) $A^{50} = \begin{bmatrix} 1 & 0 & 0 \\ 25 & 1 & 0 \\ 25 & 0 & 1 \end{bmatrix}$ (d) $A^{50} = \begin{bmatrix} 1 & 1 & 0 \\ 25 & 1 & 0 \\ 25 & 0 & 1 \end{bmatrix}$
12. If the elements of a matrix A are real positive and distinct such that $\det(A + A^T)^T = 0$ then
    (a) $\det A > 0$ (b) $\det A \geq 0$
    (c) $\det (A - A^T) > 0$ (d) $\det (AA^T) > 0$
13. If $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ and X is a non zero column matrix such that $AX = \lambda X$, where λ is a scalar, then values of λ can be
    (a) 3 (b) 6
    (c) 12 (d) 15
14. If A, B are two square matrices of same order such that $A + B = AB$ and I is identity matrix of order same as that of A, B, then
    (a) $AB = BA$ (b) $|A - I| = 0$
    (c) $|B - I| \neq 0$ (d) $|A - B| = 0$

Answers Key

Single Correct Answer Type

1. (d)	2. (c)	3. (c)	4. (d)	5. (c)
6. (d)	7. (d)	8. (b)	9. (a)	10. (d)

Multiple Correct Answers Type

11. (a, b, c)	12. (a, c, d)
13. (a, d)	14. (a, c)

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DPP 9.3

Adjoint and Inverse of Matrix

Single Correct Answer Type

1. If A is a square matrix of order 3 such that $|A| = 5$, then $|\text{Adj}(4A)| =$
   (a) $5^3 \times 4^2$ (b) $5^2 \times 4^3$ (c) $5^2 \times 16^3$ (d) $5^3 \times 16^2$
2. If A and B are two non singular matrices and both are symmetric and commute each other, then
   (a) Both $A^{-1}B$ and $A^{-1}B^{-1}$ are symmetric.
   (b) $A^{-1}B$ is symmetric but $A^{-1}B^{-1}$ is not symmetric
   (c) $A^{-1}B^{-1}$ is symmetric but $A^{-1}B$ is not symmetric
   (d) Neither $A^{-1}B$ nor $A^{-1}B^{-1}$ are symmetric
3. If A is a square matrix of order 3 such that $|A| = 2$, then $|(\text{adj } A^{-1})^{-1}|$ is
   (a) 1 (b) 2 (c) 4 (d) 8
4. Let matrix $A = \begin{bmatrix} x & y & -z \\ 1 & 2 & 3 \\ 1 & 1 & 2 \end{bmatrix}$, where $x, y, z \in N$. If $|\text{adj}(\text{adj}(\text{adj}(\text{adj } A)))| = 4^8 \cdot 5^{16}$, then the number of such (x, y, z) are
   (a) 28 (b) 36 (c) 45 (d) 55
5. A be a square matrix of order 2 with $|A| \neq 0$ such that $|A + |A|\text{ adj }(A)| = 0$, where adj$(A)$ is a adjoint of matrix A, then the value of $|A - |A|\text{ adj }(A)|$ is
   (a) 1 (b) 2 (c) 3 (d) 4
6. If A is a skew symmetric matrix, then $B = (I - A)(I + A)^{-1}$ is (where I is an identity matrix of same order as of A)
   (a) idempotent matrix (b) symmetric matrix
   (c) orthogonal matrix (d) none of these
7. If $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$, then the trace of the matrix Adj(Adj A) is
   (a) 1 (b) 2 (c) 3 (d) 4
8. If $A = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & -3 \\ 2 & 1 & 0 \end{bmatrix}$ and $B = (\text{adj } A)$ and $C = 5A$, then find the value of $\dfrac{|\text{adj } B|}{|C|}$
   (a) 25 (b) 2 (c) 1 (d) 5
9. Let A and B be two non-singular square matrices such that $B \neq I$ and $AB^2 = BA$. If $A^3 = B^{-1}A^3B^n$, then value of n is
   (a) 4 (b) 5 (c) 6 (d) 7
10. If A is an idempotent matrix satisfying $(I - 0.4A)^{-1} = I - \alpha A$ where I is the unit matrix of the same order as that of A then the value of α is
    (a) $-1/3$ (b) $1/3$ (c) $-2/3$ (d) $2/3$
11. If A and B are two non-singular matrices which commute, then $(A(A + B)^{-1}B)^{-1}(AB) =$
    (a) $A + B$ (b) $A^{-1} + B^{-1}$
    (c) $A^{-1} + B$ (d) none of these

Multiple Correct Answers Type

12. If A is a non-singular matrix of order $n \times n$ such that $3ABA^{-1} + A = 2A^{-1}BA$, then
    (a) A and B both are identity matrices
    (b) $|A + B| = 0$
    (c) $|ABA^{-1} - A^{-1}BA| = 0$
    (d) $A + B$ is not a singular matrix
13. If the matrix A and B are of 3×3 and $(I - AB)$ is invertible, then which of the following statement is/are correct?
    (a) $I - BA$ is not invertible
    (b) $I - BA$ is invertible
    (c) $I - BA$ has for its inverse $I + B(I - AB)^{-1}A$
    (d) $I - BA$ has for its inverse $I + A(I - BA)^{-1}B$
14. If A is a square matrix such that $A.(\text{Adj } A) = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$, then
    (a) $|A| = 4$ (b) $|\text{adj } A| = 16$
    (c) $\dfrac{|\text{adj}(\text{adj } A)|}{|\text{adj } A|} = 16$ (d) $|adj\ 2A| = 128$

Answers Key

Single Correct Answer Type

1. (c)	2. (a)	3. (c)	4. (b)	5. (d)
6. (c)	7. (a)	8. (c)	9. (c)	10. (c)
11. (a)				

Multiple Correct Answers Type

12. (b, c) 13. (b, c)
14. (a, b, c)

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CHAPTER

10

Probability

DPP 10.1

Probability Based on Permutation and Combination and Addition Theorem of Probability

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Single Correct Answer Type

1. If four vertices of a regular octagon are chosen at random, then the probability that the quadrilateral formed by them is a rectangle is

(a) $\frac{1}{8}$ (b) $\frac{2}{21}$

(c) $\frac{1}{32}$ (d) $\frac{1}{35}$

2. Six fair dice are thrown independently .The probability that there are exactly 2 different pairs (A pair is an ordered combination like 2, 2, 1, 3, 5, 6) is

(a) 5/72 (b) 26/72

(c) 125/144 (d) 5/36

3. If the letters of the word MATHEMATICS are arranged arbitrarily, the probability that C comes before E, E before H, H before I and I before S is

(a) $\frac{1}{75}$ (b) $\frac{1}{24}$

(c) $\frac{1}{120}$ (d) $\frac{1}{720}$

4. Two squares of 1×1 are chosen at random on a chessboard. What is the probability that they have a side in common?

(a) 1/18 (b) 64/4032

(c) 63/64 (d) 1/9

5. In a game of chance a player throws a pair of dice and scores points equal to the difference between the numbers on the two dice. Winner is the person who scores exactly 5 points more than his opponent. If two players are playing this game only one time, then the probability that neither of them wins is

(a) $\frac{1}{54}$ (b) $\frac{1}{108}$

(c) $\frac{53}{54}$ (d) $\frac{107}{108}$

6. If a and b are randomly chosen from the set {1, 2, 3, 4, 5, 6, 7, 8, 9}, then the probability that the expression $ax^4 + bx^3 + (a + 1)x^2 + bx + 1$ has positive values for all real values of x is

(a) $\frac{34}{81}$ (b) $\frac{31}{81}$

(c) $\frac{32}{81}$ (d) $\frac{10}{27}$

7. A word of at least 5 letters is made at random from 3 vowels and 3 consonants, all the letters being different. The probability that no consonant falls between any two vowels in the word is

(a) $\frac{9}{20}$ (b) $\frac{9}{10}$

(c) $\frac{7}{10}$ (d) $\frac{11}{20}$

8. Matrices of order 3×3 are formed by using the elements of the set $A = \{-3, -2, -1, 0, 1, 2, 3\}$, then probability that matrix is either symmetric or skew symmetric is

(a) $\frac{1}{7^6} + \frac{1}{7^3}$ (b) $\frac{1}{7^9} + \frac{1}{7^3} - \frac{1}{7^6}$

(c) $\frac{1}{7^3} + \frac{1}{7^9}$ (d) $\frac{1}{7^3} + \frac{1}{7^6} - \frac{1}{7^9}$

9. A box contains 10 tickets numbered from 1 to 10. Two tickets are drawn one by one without replacement. The probability that the "difference between the first drawn ticket number and the second is not less than 4" is

(a) $\frac{7}{30}$ (b) $\frac{14}{30}$

(c) $\frac{11}{30}$ (d) $\frac{10}{30}$

10. Three vertices are chosen randomly from the seven vertices of a regular 7-sided polygon. The probability that they form the vertices of an isosceles triangle is

(a) $\frac{1}{7}$ (b) $\frac{1}{3}$

(c) $\frac{3}{7}$ (d) $\frac{3}{5}$

11. Given four pair of gloves, they are distributed to four persons. Each person is given a right-handed and left-handed glove, then the probability that no person gets a pair is

(a) 3/8 (b) 5/8 (c) 1/4 (d) 3/4

12. The probability that $\sin^{-1}(\sin x) + \cos^{-1}(\cos y)$ is an integer $x, y \in \{1, 2, 3, 4\}$ is

(a) $\frac{1}{16}$ (b) $\frac{3}{16}$

(c) $\frac{15}{16}$ (d) $\frac{14}{16}$

13. Of all the mappings that can be defined from the set $A: \{1, 2, 3, 4\} \to B: \{5, 6, 7, 8, 9\}$, a mapping is randomly selected. The chance that the selected mapping is strictly monotonic is

(a) $\frac{1}{125}$ (b) $\frac{2}{125}$

(c) $\frac{3}{25}$ (d) $\frac{6}{25}$

14. 5 different balls are placed in 5 different boxes randomly. Find the probability that exactly two boxes remain empty. Given each box can hold any number of balls.

(a) $\frac{24}{125}$ (b) $\frac{12}{25}$

(c) $\frac{96}{625}$ (d) None of these

15. 10 different books and 2 different pens are given to 3 boys so that each gets equal number of things. The probability that the same boy does not receive both the pens is

(a) 5/11 (b) 7/11 (c) 9/11 (d) 6/11

16. Let a function $f: X \to Y$ is defined where $X = \{0, 1, 2, 3, ..., 9\}$, $Y = \{0, 1, 2, ..., 100\}$ and $f(5) = 5$, then the probability that the function of type $f: x \to B$ where $B \subseteq Y$ is of bijective in nature is

(a) $\frac{10!}{\sum_{r=1}^{101} r^9 \cdot {}^{100}C_{r-1}}$ (b) $\frac{{}^{101}C_9 \cdot 9!}{\sum_{r=1}^{101} r^{10} \cdot {}^{100}C_r}$

(c) $\frac{{}^{100}C_9 \cdot 9!}{\sum_{r=1}^{101} r^{10} \cdot {}^{101}C_r}$ (d) $\frac{{}^{100}C_9 \cdot 9!}{\sum_{r=1}^{101} r^9 \cdot {}^{100}C_{r-1}}$

17. Two distinct numbers a and b are chosen randomly from the set $\{2, 2^2, 2^3, \ldots 2^{25}\}$. Then the probability that $\log_a b$ is an integer is

(a) $\frac{131}{300}$ (b) $\frac{31}{300}$

(c) $\frac{21}{200}$ (d) $\frac{62}{300}$

18. Given that $x \in [0, 1]$ and $y \in [0, 1]$. Let A be the event of selecting a point (x, y) satisfying $y^2 \geq x$ and B be the event selecting a point (x, y) satisfying $x^2 \geq y$, then

(a) $P(A \cap B) = \frac{1}{3}$ (b) $A \subset B$

(c) $2P(A) = 3P(B)$ (d) $P(B) < P(A)$

19. A and B are 2 events such that $P(A) = \frac{3}{4}$ and $P(B) = \frac{5}{8}$. If x and y are the possible minimum and maximum values of $P(A \cap B)$, then the value of $a + b$ is

(a) 0.5 (b) 0.8 (c) 0.9 (d) 1

20. If A and B are two events such that $P(A \cap B) = 0.3$ and $P(A' \cap B') = 0.6$, then the value of $P(A \cap B'$ or $A' \cap B)$ is equal to

(a) 0.9 (b) 0.7 (c) 0.3 (d) 0.1

21. The probability that a dealer will sell at least 20 TV sets during a day is 0.45 and the probability that he will sell less than 24 TV sets is 0.74. The probability that he will sell 20, 21, 22 or 23 TV sets during the day is

(a) 0.19 (b) 0.29 (c) 0.333 (d) 0.81

22. Consider the Cartesian plane R^2 and let X denote the subset of points for which both coordinates are integer. A coin of diameter 1/2 is tossed randomly onto the plane. The probability p that the coin covers a point of X

(a) 0.2 (b) 0.8

(c) 1.2 (d) None of these

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Multiple Correct Answers Type

23. The probabilities of events, $A \cap B$, A, B and $A \cup B$ are respectively in A.P. with second term equal to the common difference. Therefore A and B are
(a) mutually exclusive
(b) independent
(c) such that one of them must occur
(d) such that one is twice as likely as the other

24. 5 players of equal strength play one each with each other. $P(A)$ = probability that at least one player wins all matches he (they) play. $P(B)$ = probability that at least one player losses all his (their) matches.

(a) $P(A)=\frac{5}{16}$ (b) $P(B)=\frac{7}{16}$

(c) $P(A\cap B)=\frac{5}{32}$ (d) $P(A\cup B)=\frac{15}{32}$

25. If A and B are exhaustive events in a sample space such that probabilities of the events $A \cap B$, A, B and $A \cup B$ are in A.P. If $P(A) = K$, where $0 < K \leq 1$, then

(a) $P(B)=\frac{K+1}{2}$ (b) $P(A\cap B)=\frac{3K-1}{2}$

(c) $P(A \cup B) = 1$ (d) $P(A'\cup B')=\frac{3(1-K)}{2}$

26. A boy has a collection of blue and green marbles. The number of blue marbles belong to the set $\{2, 3, 4, \ldots, 13\}$. If two marbles are chosen simultaneously and at random from his collection, then the probability that they have different colours is $\frac{1}{2}$. Possible number of blue marbles is

(a) 3 (b) 6 (c) 10 (d) 12

27. A fair coin is tossed n times. Let X denote the number of times head occurs. If $P(X = 4)$, $P(X = 5)$ and $P(X = 6)$ are in arithmetic progression, then the value of n can be
(a) 14 (b) 12 (c) 10 (d) 7

28. If $A_1, A_2, \ldots\ldots, A_n$ be any events of the same sample space then

(a) $\sum_{i=1}^{n} P(A_i) = 1$

(b) $\sum P(A_i) \leq 1$ if $A_1, A_2, \ldots\ldots, A_n$ are disjoint

(c) $\sum P(A_i) \geq 1$ if $A_1, A_2, \ldots\ldots, A_n$ are exhaustive events

(d) None of these

Answers Key

Single Correct Answer Type

1. (d)	2. (b)	3. (c)	4. (a)	5. (c)
6. (c)	7. (d)	8. (d)	9. (a)	10. (d)
11. (a)	12. (b)	13. (b)	14. (b)	15. (c)
16. (d)	17. (b)	18. (a)	19. (d)	20. (d)
21. (a)	22. (a)			

Multiple Correct Answers Type

23. (a, d)	24. (a, c, d)
25. (a, b, c, d)	26. (a, b, c)
27. (a, d)	28. (b, c)

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DPP 10.2

Probability of Independent Events

Single Correct Answer Type

1. A die is thrown 31 times. The probability of getting 2, 4, or 5 at most 15 times is

(a) $\frac{1}{3}$ (b) $\frac{1}{4}$

(c) $\frac{1}{5}$ (d) $\frac{1}{2}$

2. The records of a hospital show that 10% of the cases of a certain disease are fatal. If 6 patients are suffering from the disease, then the probability that only three will die is

(a) 1458×10^{-5} (b) 1458×10^{-6}

(c) 41×10^{-6} (d) 8748×10^{-5}

3. The probabilities of A, B, and C solving a problem independently are respectively $\frac{1}{4}, \frac{1}{5}, \frac{1}{6}$. If 21 such problems are given to A, B, and C then the probability that at least 11 problems can be solved by them is

(a) ${}^{21}C_{11}\left(\frac{1}{2}\right)^{11}$ (b) $\frac{1}{2}$

(c) $\left(\frac{1}{2}\right)^{11}$ (d) None of these

4. A fair coin is tossed until one of the two sides occurs twice in a row. The probability that the number of tosses required is even is

(a) 1/3 (b) 2/3 (c) 1/4 (d) 3/4

5. A man throws a die until he gets a number greater than 3. The probability that he gets 5 in the last throw

(a) 1/3 (b) 1/4 (c) 1/6 (d) 1/36

6. Suppose A and B are two events with $P(A) = 0.5$ and $P(A \cup B) = 0.8$. Let $P(B) = p$ if A and B are mutually exclusive and $P(B) = q$ if A and B are independent, then

(a) $p = q$ (b) $p = 2q$

(c) $2p = q$ (d) $p + q = 1$

7. A biased coin with probability $p(0 < p < 1)$ of falling tails is tossed until a tail appears for the first time. If the probability that tail comes in odd number of trials is $\frac{2}{3}$, then p equals

(a) $\frac{1}{4}$ (b) $\frac{1}{3}$ (c) $\frac{3}{4}$ (d) $\frac{1}{2}$

8. Assume that the birth of a boy or girl to a couple to be equally likely, mutually exclusive, exhaustive and independent of the other children in the family. For a couple having 6 children, the probability that their "three oldest are boys" is

(a) $\frac{20}{64}$ (b) $\frac{1}{64}$

(c) $\frac{8}{64}$ (d) none of these

9. Two persons A and B get together once a week to play a game. They always play 4 games. From past experience Mr. A wins 2 of the 4 games just as often as he wins 3 of the 4 games. If Mr. A does not always wins or always loose, then the probability that Mr. A wins any one game is (Given the probability of A's wining a game is a non-zero constant less than one).

(a) 0.5 (b) 0.6 (c) 0.8 (d) 0.9

10. Suppose A and B shoot independently until each hits his target. They have probabilities $\frac{3}{5}$, and $\frac{5}{7}$ of hitting the target at each shoot. Then the probability that B will require more shots than A is

(a) 5/21 (b) 6/31

(c) 7/41 (d) none of these

11. A fair die is tossed repeatedly. A wins if it is 1 or 2 on two consecutive tosses and B wins if it is 3, 4, 5 or 6 on two consecutive tosses. The probability that A wins if the die is tossed indefinitely, is

(a) $\frac{1}{3}$ (b) $\frac{1}{4}$ (c) $\frac{5}{21}$ (d) $\frac{2}{5}$

Multiple Correct Answers Type

12. A family has three children. Event 'A' is that family has at most one boy, Event 'B' is that family has at least one boy and one girl, Event 'C' is that the family has at most one girl. Then

(a) Events 'A' and 'B' are independent

(b) Events 'A' and 'B' are not independent

(c) Events A, B, C are independent

(d) Events A, B, C are not independent

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13. A certain coin is tossed with probability of showing head being 'p'. Let 'q' denotes the probability that when the coin is tossed four times the number of heads obtained is even. Then

(a) there is no value of p, if $q = \frac{1}{4}$

(b) there is exactly one value of p if $q = \frac{3}{4}$

(c) there are exactly two values of p if $q = \frac{3}{5}$

(d) there are exactly four values of p if $q = \frac{4}{5}$

14. A bag contains four tickets marked with numbers 112, 121, 211, and 222. One ticket is drawn at random from the bag. Let $E_i (i = 1, 2, 3)$ denote the event that ith digit on the ticket is 2. Then

(a) E_1 and E_2 are independent
(b) E_2 and E_3 are independent
(c) E_3 and E_1 are independent
(d) E_1, E_2, E_3 are independent

Answers Key

Single Correct Answer Type

1. (d) **2.** (a) **3.** (b) **4.** (b) **5.** (a)
6. (c) **7.** (d) **8.** (c) **9.** (b) **10.** (b)
11. (c)

Multiple Correct Answers Type

12. (a, d) **13.** (a, c) **14.** (a, b, c)

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DPP 10.3

Probability of Dependent Events, Total Probability Theorem and Bayes' Theorem

Single Correct Answer Type

1. For two events A and B, if $P(A) = P\left(\frac{A}{B}\right) = \frac{1}{4}$ and $P\left(\frac{B}{A}\right) = \frac{1}{2}$, then which of the following is not true?
   (a) A and B are independent
   (b) $P\left(\frac{A'}{B}\right) = \frac{3}{4}$
   (c) $P\left(\frac{B'}{A'}\right) = \frac{1}{2}$
   (d) None of these
2. A and B are events of an experiment such that $0 < P(A)$, $P(B) < 1$. If $P(B') > P(A')$, then
   (a) $P(A \cap B') < P(A' \cap B)$
   (b) $P(A \cap B') = P(A' \cap B)$
   (c) $P(B/A) < P(A/B)$
   (d) $P(B/A) > P(A/B)$
3. If A and B are two events such that $P(A) = 0.3$, $P(B) = 0.25$, $P(A \cap B) = 0.2$, then $P\left(\left(\frac{A^C}{B^C}\right)^C\right)$ is equal to
   (a) $\frac{2}{15}$ (b) $\frac{11}{15}$ (c) $\frac{13}{15}$ (d) $\frac{14}{15}$
4. A number is selected at random from the first twenty-five natural numbers. If it is a composite number, then it is divided by 5. But if it is not a composite number, it is divided by 2. The probability that there will be no remainder in the division is
   (a) $\frac{11}{30}$ (b) 0.4 (c) 0.2 (d) none of these
5. If two events A and B such that $P(A') = 0.3$, $P(B) = 0.5$ and $P(A \cap B) = 0.3$, then $P(B/A \cup B')$ is
   (a) 3/8 (b) 2/3 (c) 5/6 (d) 1/4
6. In a hurdle race, a runner has probability p of jumping over a specific hurdle. Given that in 5 trials, the runner succeeded 3 times, the conditional probability that the runner had succeeded in the first trial, is
   (a) 3/5 (b) 2/5
   (c) 1/5 (d) None of these
7. A box contains 4 white and 3 black balls. Another box contains 3 white and 4 black balls. A die is thrown. If it exhibits a number ≤ 3, the ball is drawn from the first box. Otherwise, a ball is drawn from the second box. A ball drawn is found to be black. The probability that it has been drawn from the second box is
   (a) 3/7 (b) 4/7 (c) 6/17 (d) 8/17
8. The probabilities of solving a problem correctly by A and B are $\frac{1}{8}$ and $\frac{1}{12}$ respectively. Given that they obtain the same answer after solving a problem and the probability of a common mistake by them is $\frac{1}{1001}$, then probability that their solution is correct is (Assuming that if they commit different mistake, then their answers will differ)
   (a) $\frac{77}{96}$ (b) $\frac{14}{15}$ (c) $\frac{2}{5}$ (d) $\frac{13}{14}$
9. The probability of event A is 3/4. The probability of event B, given that event A occurs is 1/4. The probability of event A, given that event B occurs is 2/3. The probability that neither event occurs is
   (a) $\frac{1}{6}$ (b) $\frac{27}{112}$ (c) $\frac{5}{32}$ (d) $\frac{1}{8}$
10. An urn contains three white, six red and four black balls. Two balls are selected at random. What is the probability that one ball is red and other is white, given that they are of different colour?
    (a) $\frac{2}{3}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) none of these
11. Let A, B, C be 3 events such that $P(A/B) = \frac{1}{5}$, $P(B) = \frac{1}{2}$, $P(A/C) = \frac{2}{7}$ and $P(C) = \frac{1}{2}$, then $P(B/A)$ is
    (a) $\frac{4}{11}$ (b) $\frac{5}{11}$ (c) $\frac{6}{11}$ (d) $\frac{7}{17}$
12. A coin is tossed. If head appears a fair die is thrown three times otherwise a biased die with probability of obtaining an even number twice as that of an odd number is thrown three times. If (n_1, n_2, n_3) is an outcome, $(1 \leq n_1 \leq 6)$ and is found to satisfy the equation $i^{n_1} + i^{n_2} + i^{n_3} = 1$, , then the probability that a fair die was thrown is (where $i = \sqrt{-1}$)

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(a) $\frac{1}{12}$ (b) $\frac{1}{3}$

(c) $\frac{27}{59}$ (d) None of these

13. For any events A and B. Given $P(A \cup B) = 0.6$, $P(A) = P(B)$, $P(B/A) = 0.8$. Then the value of $P[(A \cap \overline{B}) \cup (\overline{A} \cap B)]$ is
(a) 1/3 (b) 1/2 (c) 1/4 (d) 1/5

Multiple Correct Answers Type

14. Mohan post a letter to Sohan. It is known that one letter out of 10 letters do not reach its destination. If it is certain that Sohan will reply if he receives the letter. If A denotes the event that the Sohan receives the letter and B denotes the event that Mohan gets a reply, then

(a) $P(B) = \frac{81}{100}$ (b) $P(A \cap B) = \frac{81}{100}$

(c) $P\left(\frac{A}{\overline{B}}\right) = \frac{9}{19}$ (d) $P(A \cup B) = \frac{9}{10}$

15. If E_1 and E_2 are two events such that $P(E_1) = 1/4$, $P(E_2/E_1) = 1/2$ and $P(E_1/E_2) = 1/4$, then
(a) then E_1 and E_2 are independent
(b) E_1 and E_2 are exhaustive
(c) E_2 is twice as likely to occur as E_1
(d) Probabilities of the events $E_1 \cap E_2$, E_1 and E_2 are in G.P.

16. For $P(A) = \frac{3}{8}$; $P(B) = \frac{1}{2}$; $P(A \cup B) = \frac{5}{8}$ which of the following is/are true?
(a) $P(A^C/B) = 2P(A/B^C)$
(b) $P(B) = P(A/B)$
(c) $15P(A^C/B^C) = 8P(B/A^C)$
(d) $P(A/B^C) = (A \cap B)$

17. Consider the word POSSIBILITY. In the arrangement of the letters of the above word let A and B denote the event that the 2 S's are together and the 3 I's are together respectively, then
(a) $P(A) = P(B) = \frac{3}{11}$ (b) $P(A \cap B) = \frac{2}{165}$
(c) $P(A \cup B) = \frac{7}{31}$ (d) $P(B/A) = \frac{1}{15}$

Comprehension Type

For Q. 18 and 19

A slip of paper is given to a person A who marks it either with a plus sign or a minus sign. The probability of his writing a plus sign is 1/3. A passes the slip to B, who may either leave it alone or change the sign before passing it to C. Next C passes the slip to D after perhaps changing the sign. Finally D passes it to a referee after perhaps changing the sign. B, C, D each change the sign with probability 2/3.

18. The probability that the referee observes a plus sign on the slip if it is known that A wrote a plus sign is
(a) 14/27 (b) 16/27 (c) 13/27 (d) 17/27

19. If the referee observes a plus sign on the slip then the probability that A originally wrote a plus sign is
(a) 13/41 (b) 19/27 (c) 17/25 (d) 21/37

Answers Key

Single Correct Answer Type

1. (d)	2. (c)	3. (a)	4. (c)	5. (a)
6. (a)	7. (b)	8. (d)	9. (c)	10. (b)
11. (d)	12. (c)	13. (d)		

Multiple Correct Answers Type

14. (a, b, c, d) 15. (a, c, d)
16. (a, b, c, d) 17. (b, d)

Comprehension Type

18. (c) 19. (a)

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